A Manual for Teaching Basic Math to Adults

Changing the Way We Teach Math

Kate Nonesuch

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“Changing the way we teach math—that would be as hard as turning a BC ferry around.” — Bob Darling, ABE math instructor

Introduction

I wrote this manual for ABE math instructors who, like me, are interested in changing their teaching practice to bring it more in line with recommendations from the research literature on teaching numeracy to adults. I wrote it after consulting with more than 100 people in British Columbia about their practice, and I wrote it to address the concerns they expressed to me at those consultations. The manual sets out some “best practices” from the literature, then outlines some difficulties instructors may face in implementing them, and makes suggestions for overcoming the difficulties. Finally, there are many pages of activities ready for immediate classroom use, which provide examples of some ways of implementing the best practices.

Chapter 1 describes the process I engaged in to consulting the literature and the field. When I consulted instructors, they said they were most interested in increasing their use of four strategies for teaching math: hands-on learning, group work, using real-life situations for teaching math, and giving students responsibility for their learning. They said that student resistance and time constraints figured large among the barriers they faced to using those strategies.

Chapter 2 finds us facing up squarely to the first of those barriers: resistance to changing traditional math practice. Since we cannot deal with resistance unless we deal with emotions, Chapter 3 talks about emotions in teaching and learning math.

Those two topics out of the way, Chapter 4 deals with hands-on learning, using manipulatives and visual and practical presentations to teach math concepts; Chapter 5 gives many examples of group activities that can be used with students who don’t have well-developed group skills; Chapter 6 takes on the questions of real-life math, building on the basis of strategies in the previous chapters; and Chapter 7 goes on to discuss strategies for putting students in charge of their own learning.

Finally, in Chapter 8, there are activities, requiring little or no preparation, which instructors can use for various content areas of basic math. These activities are examples of how strategies discussed in the first seven chapters can be implemented, and, in addition, respond to the second barrier instructors mentioned, that is, lack of time to prepare material.

How did this manual come to be?

This manual is part of a project funded by the National Office of Literacy and Learning (NOLL). My research question was “How can ABE math instructors in BC apply research findings to their practice?” While the question was big, and each of its many parts was big, it came from a personal question: How can I apply research findings to my own teaching practice? I have been teaching for more than 15 years at Malaspina University-College, at the Cowichan Campus in Duncan. Mainly I have taught at the fundamental level, with occasional forays into introductory algebra. During that time, and based on previous teaching experience, I tried many different methods and texts in my attempt to teach so that students would understand math and, I hoped, learn to like it. During that period, being too busy teaching, I didn’t have time to search systematically for relevant research material, and it wasn’t until the recent explosion of material online that searching from a remote location became at all feasible. Hence, when I received funding for this project, I welcomed the luxury of spending time away from the classroom, to read, think, talk, and write about teaching math.

I started by looking for research findings on improving instruction of basic math to adult students; as I read the research I found, and thought about it in relation to my own teaching experience and philosophy, I began to shape the consultation I would have with other instructors. I wondered if others would have the same reaction as I did to some of the research findings. I wondered if they would be as interested as I was in improving the way basic math is taught to adults in BC.

My reading was followed by face-to-face group consultations with 90 people (mainly instructors) interested in teaching basic math to adults in BC, and by two on-line conferences with more than 100 literacy practitioners from BC, other parts of Canada, the USA, Australia and the United Kingdom. However, the face-to-face consultations were more focused, and resulted in a clearer sense of direction as to what instructors were interested in. The on-line discussions were wide-ranging, but, perhaps due to the nature of the on-line setting, or to my lack of expertise, or to the widely differing situations and assumptions that participants had, I did not get the kind of specific direction for the manual that came from the face-to-face consultations. Although many of the issues raised were the same in both places, and the data did not contradict each other, I got clearer answers at the face-to-face consultations about what changes in practice instructors were interested in making. Moreover, the face-to-face consultations were with instructors teaching in BC, under current conditions. Consequently, I have based the contents of this manual mainly on the data I collected at the face-to-face consultations, which I will explain in detail a bit further on.

After each of the consultations, based on discussions and questions at the session, I sent out activities to use with students, or further sources of information, to workshop participants. That material is included in this manual, in a revised form. Marina Niks, RiPAL BC, commented on a draft of this chapter, and practitioners Evelyn Battell and Karen Burns read an early first draft of this manual, and I revised it in the light of their comments and suggestions. Bob Darling, Leslie Kiehlbauch, Dee McRae, Vicki Noonan and Iris Strong read the next draft, which I revised to take the final shape you see here.

The Literature

I began by searching out and reading the research about teaching basic math to adults. I concentrated on books and articles about teaching basic concepts (whole number operations, common fractions, decimals and percent) to the kinds of ABE/Literacy students who come to programs in BC. Even though I was looking at programs in BC, I wanted to see what was going on in other places, so I read some literature from Australia, the UK and the USA, as well as Canada. I concentrated on material about instructional methods and teaching philosophy, rather than questions about policy or curriculum.

Although some major players, such as Diana Coben (2003) from the British National Research and Development Centre for Adult Literacy and Numeracy, have said that there hasn't been enough research done to make any definitive statements about how to improve instruction, many people have gone ahead to make many definitive statements about exactly that. I soon found a comprehensive and concise statement of principles written by Lynda Ginsburg and Iddo Gal (1996) that fit well with what many authors were saying:

1. Determine what learners already know about a topic before instruction.
2. Address and evaluate attitudes and beliefs regarding both learning math and using math.
3. Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations and hands-on activities.
4. Encourage the development and practice of estimation skills.
5. Emphasize the use of "mental math" and the need to connect different mathematical skills and concepts.
6. View computation as a tool for problem solving, not an end in itself; encourage use of multiple solution strategies.
7. Develop learners' calculator skills and foster familiarity with computer technology.
8. Provide opportunities for group work.
9. Link numeracy and literacy instruction by providing opportunities for students to communicate about math.
10. Situate problem-solving tasks within familiar, meaningful, realistic contexts in order to facilitate transfer of learning.
11. Develop learners' skills in interpreting numerical or graphical information appearing within documents and text.
12. Assess a broad range of skills, reasoning processes and dispositions, using diverse methods (Ginsburg & Gal, 1996, p 2, ff).

Further, I found two recommendations about best practices for retention of students that I thought fit well into the topic, from a report prepared for the College Sector Committee for Adult Upgrading, a committee of the Association of Colleges of Applied Arts and Technology of Ontario:

- The management of learning should lead to the development and reinforcement of the learner's control of his/her own learning.
- The management of learning should strive to build a shared power relationship in the classroom that contributes to self-esteem and self-confidence ("Best practices in managing the classroom to improve student commitment", no date).

I was interested in these final two best practices because they fit with my own ideas about teaching adults; in recent years I have thought and written about the power relationships between teacher and learner, and I have worked hard at the Reading and Writing Centre to facilitate learner control of learning. The two recommendations from *Best practices in managing the classroom to improve student commitment* deal with the "heartwork" of learning math, rather than the headwork. In my opinion, some of the difficulty with learning and teaching math is a neglect of the emotional or affective side of teaching and learning. The head that comes to learn math is attached to a heart, and cannot function except as the heart influences how much and what kind of work the head can do. Similarly, the head that comes to teach math is attached to a heart, and functions only in relationship to that heart, no matter how hard the head tries to deny or bury the emotions.

Reaction to the Literature

At first when I found such recommendations for instruction, such articulations of "best practices," I was taken aback. Perhaps I had been looking for a magic bullet, but I was surprised to find nothing very new. I had heard of all of these things over the years I had been teaching math to adults, and I agreed with all of them. Yet, if that was so, why did I not apply them in my teaching practice? I had certainly tried to apply many of them, perhaps even all of them, from time to time, but had been unable to make them work consistently for me. In spite of my best intentions and in spite of all the work I had done to make my teaching successful, I had often fallen back on teaching the way I was taught, tying my instruction to a text that I didn't particularly like, and focusing on getting students through the material rather than teaching for understanding.

Furthermore, I didn't know any other math instructor who was able to consistently use all of these "best practices," although I knew many who were regarded by their students and colleagues as good teachers, and who spent many hours outside of class working to improve their teaching. They, like me, had tried some or all of the strategies suggested by the experts, but were not able to consistently put them all into practice.

More literature

So I began to look for more literature that might explain why this state of affairs existed. Why is it so difficult to apply these best practices in math classes? What are the barriers teachers face when they introduce them into their classes? For a description of my search and what I found, see my literature review, “More Complicated Than It Seems: A Review of Literature on Adult Numeracy Instruction” (Nonesuch, forthcoming).

More reactions: Consultations with the field

I took my dilemma with the research to eight groups of people in BC who are concerned with teaching math. I sent out an invitation to do a workshop in which I would outline some of the research findings, and consult with participants about their reaction and their practice. Many groups of instructors were interested in participating, and in the end I did eight workshops: with the Fundamentals Articulation Committee, with the Math Articulation Committee, with three groups of ABE math instructors and tutors from three different colleges, and with two other more mixed groups, which included people who were instructors at colleges and school district programs, university faculty, tutors, and staff of community-based programs; a further workshop included people with a wider range of interest in math, including librarians and parents as well as math instructors. The groups varied in size from 7 to 21 people; in total, 90 attended the 8 workshops.

At each workshop, I began by presenting some of the research I had read, always including in some detail the best practices cited above, some of the findings in support of those practices and some material showing where some of them might be misguided or difficult to implement, along with some examples from my own experience.

First, I asked participants to indicate which of the best practices they had never heard of before, and almost without fail, they said they had heard of them all. Occasionally a participant in the workshop who was not an ABE math instructor—perhaps a new tutor or an administrator—would say they were unfamiliar with one or two, but generally they all had the same reaction that I had had—“I have heard of these before.” The only one that I heard any disagreement with at this preliminary stage of the workshop was the recommendation about the use of calculators. This recommendation was controversial in the true sense of the word—some instructors disagree with any use of calculators at the basic math level, while others agree with the recommendation and teach appropriate use in their classes.

After this initial response, I asked participants for a more detailed response to the research, and particularly asked participants what made the recommendations difficult to apply in their practice. Following this group discussion, participants responded in writing in two ways: creatively, in small groups, and more matter-of-factly as individuals.

Recipes and Equations

At the workshops, I asked participants to choose between two activities:

- Write some equations or draw some diagrams for getting results in a math class and/or develop an algorithm for change in math instruction, OR
- Write a recipe for getting results or making change in a math class, giving ingredients, method, cooking time, etc.

I asked them to go to one side of the room if they wanted to write a recipe and the other side if they were interested in writing equations/diagrams/algorithms. After they separated into these two large groups, I asked them to divide themselves into smaller groups to do the activity, or to work alone if they wished. Only one person chose to work alone; the others worked mostly in groups of two or three, occasionally in a group of four or five. Many of the groups using math symbols made several equations, or elaborate flow charts. As a whole, the recipes, equations, and diagrams take up the importance of various intellectual aspects of math instruction equally with the emotional aspects.

Response to Questions

At each workshop I gave participants a sheet of questions: What did you hear that you didn't already know about? What did you hear that confirms your own experience? What seems to go against your own experience? What findings would you most like to implement in your own situation? If/when you try to implement these findings what gets in your way? Eighty people turned in these questionnaires to me at the end of the workshops; I discarded answers from four people who did not teach math (from the Fundamentals Articulation Committee, which includes both English and Math instructors) and who indicated on their sheets that they were not math teachers.

As I thought about writing this manual, I was especially interested in the answers to the final two questions: "What findings would you most like to implement in your own situation?" and "If/when you try to implement these findings what gets in your way?" I knew that the answers to the first question would guide me about what to write about, and the answers to the second would shape what I had to say on the topic.

"What findings would you most like to implement?"

Strategy	*Number of instructors interested in adopting the strategy*	*Barrier: Time constraints*	*Barrier: Student resistance*	*Barrier: Instructor's lack of training or discomfort*	*Barrier: Rigid curriculum*
Use concrete or visual activities	**19**	**10**	**7**	**6**	
Situate problem-solving tasks within familiar contexts	**15**	**3**	**5**		**3**
Provide opportunities for group work	**11**	**6**	**6**		**2**
Develop shared power relationship in the classroom	**12**	**2**	**4**	**3**	

Table 1 Barriers to implementing particular strategies

Of the 56 participants who answered this question, some of them said they would like to implement more than one finding. As Table 1 shows, four strategies were overwhelmingly chosen by participants:

- Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations and hands-on activities.

- Situate problem-solving tasks within familiar, meaningful, realistic contexts in order to facilitate transfer of learning.
- The management of learning should lead to the development and reinforcement of the learner's control of his/her own learning **and** the management of learning should strive to build a shared power relationship in the classroom that contributes to self-esteem and self-confidence. (I combined two strategies, both dealing with sharing power/responsibility in classroom management.)
- Provide opportunities for group work.

My notes from the discussions at the workshop confirm that these were the concerns most articulated by participants, and they were also echoed in the equations and recipes that the participants produced.

If/when you try to implement these findings what gets in your way?
Very interesting things happened when participants wrote down the barriers they experienced in implementing these strategies. The same two barriers were cited for every strategy—student resistance and time constraints. In three of the four strategies, student resistance was the most frequently cited barrier! The most frequent barriers mentioned are shown in Table 1, above.

Conclusion

I was surprised a little about the answers to the questions. I was surprised that so many instructors said that student resistance was a major factor. I had talked in the workshops about my own experience with student resistance, but it seems I opened a floodgate for teachers to say how much it affected them. I was surprised that so many instructors said they wanted to work with manipulatives or graphic representations, but were unwilling to do so because they were uncomfortable working with them themselves. One group developed a recipe which was in essence a plan for a workshop to help them get more comfortable with using manipulatives, and I received two requests after the workshop to come and do some professional development with instructors on the subject. I was surprised that participants wanted to connect math with real life, since I had presented some research findings that showed how difficult it was to do so (for example, an article by Alison Tomlin (2002); I thought it showed the hunger participants felt to make math meaningful to students, and I was delighted, if surprised, to find it showing up.

Based on the findings from the workshops, I decided that in this manual I would write about implementing those four "best practices" that participants had said they were most interested in: hands-on learning, using real-life problems to teach math, using group work as a teaching technique, and helping learners take control of and responsibility for their own learning. Further, I knew that I wanted to write about overcoming the barriers instructors named to implementing these best practices, rather than simply saying, "Do this, do that." I wanted to lay out the complexities that lay behind those deceptively simple statements of how to improve math instruction. Much of the art of teaching is hidden, not only to people who have nothing to do with education, but also to teachers themselves, to administrators and to students. So much of what we do well is not articulated, and so much of what is difficult to do is also not articulated, so that it seems that we do not do it well because of ignorance, or inattention, rather than the complexity of the factors that influence our work. By laying out the complexities, I hope to make clear the problems that face us in implementing these best practices. Only once the problems are clear can we go about finding solutions.

I knew from the beginning that I wanted to make this manual as practical as possible, and, given the number of people who referred to a lack of time as a barrier to implementing the strategies, the need for activities that could be used in the classroom with little or no further preparation was underscored. I have gathered the material to use with students presented here from many sources—from my own experience, from material developed by other practitioners that I have used myself, and, finally, I have developed ideas suggested at the workshops and from print sources.

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If you truly respect students, you will honour their resistance. You have to set aside that niggling belief that they'll learn it if you can only force them to take part.
— Evelyn Battell, ABE Instructor

Resistance

When I began teaching basic math (whole numbers, fractions, decimals and percent) to adults twenty-five years ago, I started teaching as I had been taught, that is, the teacher did the math at the blackboard and the students watched the teacher do math, and listened to her talk about doing it. Then they worked in their own books, and took the tests at the end of the chapter and at the end of term. From the beginning, from the very first term, I knew it wouldn't work. Students were bored and frustrated by their lack of activity and their lack of understanding. I was bored and frustrated by their lack of engagement and their lack of understanding. I wanted more.

I began to change my teaching practice in a variety of ways—more emphasis on teaching concepts rather than algorithms; more group work, less lecture time; more emphasis on students discovering patterns; more emphasis on math thinking and problem solving; and more use of real life problems and a greater reliance on manipulatives and models. Indeed, some of the very things that I found in the literature when I went searching recently.

When I began to change my teaching practice, I met resistance on all fronts—my own resistance, students' resistance, and resistance from the programs I worked in over the years. Overcoming this resistance to new methods became the first hurdle to changing my practice and feeling comfortable with the changes.

My Own Resistance

As I changed my ways of teaching to include more student participation, problem solving, math thinking, group work, and use of manipulatives and models, a voice inside my head kept talking to me.

What you are doing is not real math, I could hear the voice say. It is frivolous to ask students to make charts and diagrams, more like art than math. Asking them to talk about their own ways of figuring things out is a tangent—you know the most efficient ways to do the calculations, so just teach them those and forget about their folk math. And getting them to work in groups to figure things out is a complete waste of time; it takes too long, and there is no way for you to tell what they are really learning in those groups. Probably they are not learning anything, and you have no control!

These new things will not help students pass the exam, the voice went on, and you know how important the exam is. You know what is on the test; drill on those things, and forget about the rest. You will just confuse the students more when you expect them to understand math rather than memorize, it continued. Just give up these new ideas, and it will be easier on everyone.

Working in these new ways just causes problems, I heard the voice say again. You know that your students have huge holes in their math backgrounds. When you stray off the beaten path in math class, you fall into one of those holes, and when you try to fill it in, you fall into another hole, and you can never get out. Furthermore, every student seems to have a different set of holes! The only way to get through the material is to stick to the path, and help the students figure out how to stay on the path to the end of the course. You know you're creating more holes by doing it this way, but it's the only way.

I dealt with the voice in various ways: I talked back to it, I found support in other teachers (both at home and away), I talked to my students, I made change slowly so I could see what was going on, and I stuck with it. I still hear that voice from time to time, but it is weaker, and I know how to make it quiet now—I point to the positive results I get when I ignore it!

Student Resistance

Some students throw themselves into school with such positive attitudes that I don't have to worry about their resistance—I only have to make sure that they have successful experiences in class to maintain their enthusiasm. Many students, however, are less open to new strategies for learning math; their responses range from silent withdrawal, to questioning their value, to open refusal to use them. Over the years, I have used different strategies to honour student resistance and work with it rather than against it. I find that students need to be able to express their resistance in order to maintain their sense of self in the class, and that when they can do so with dignity, they are more likely to be able to stay present and attend to the work. When Arleen Pare (1994) did some research for her MA thesis in my classroom, she found a positive correlation between student expression of resistance and student retention. The more complex and open their resistance to me and my teaching, the more likely they were to continue to come regularly:

> These results suggest a positive association between conscious, active resistance and regular attendance. It also suggests that the more that conscious resistance is encouraged, the more likely it is that regular attendance will result (Pare, p. 115).

As an example, take the student who keeps his coat on, sits silently at the back of the room, or near the door, and whose body language says, "I'm not here." Pare found that this student is more likely to drop out than the student who says, "Why do we have to do this stuff anyway?" and then gets a response that respects his resistance.

What Does Student Resistance Look Like?

Students sometimes express their resistance to participatory methods by simply dropping out of the class, but over the years, I have developed a teaching stance that recognizes, honours, and encourages open expression of their resistance. As a result, many students will question my methods when they are new to them. As you will see from the examples that follow, their resistance may be indirect, and often comes in the form of a question that is not a real question.

"This is not real math."

Nearly every student who enrolls in a basic math class has years of (unsuccessful) experience as a math student; it stands to reason that they have a firm idea of what math class should be and what success in math looks like. They expect me to give them sheets of questions and some tricks to help them remember how to work with fractions. When I ask them to work with manipulatives or visuals, do group activities or field trips, they resist. "This is not real math."

I deal with that resistance by acknowledging that what I am asking them to do is not what they are used to, and it feels strange. I ask them to tell me all the ways they have tried to learn math in the past. Then I ask, "Does anyone know a way to learn math that really works?" Invariably, nobody does because they have all been previously unsuccessful. This conversation with students is part of making my work and theory transparent, and makes them partners in designing their own learning. The discussion about past methods of learning math, an evaluation of what parts were more useful or less useful and the conclusion that something new needs to be tried, means that they are part of the team talking about what form teaching will take.

"How can those things help me learn?"

Most recently, I have been fortunate to work at the Reading and Writing Centre in Duncan, a storefront literacy program that gives learners a great deal of control over how the program runs. Instructors work towards making the learning process more transparent so that students can make decisions on how best to accomplish their academic goals; we ask students to be in control of their learning and to understand their own ways of learning best; and we offer a variety of modalities to students in every subject area. I often do some work with students on learning styles, and help them figure out what their best style is, as well as looking at multiple intelligences and discovering their strengths.

Then I can answer that question, or ask the student to answer it, based on his knowledge of his strengths. Students get a chance to articulate how group work helps ("Talking about what I'm doing helps me learn,") or how manipulatives help ("I'm a body [kinesthetic] learner, and holding the pieces and piling them up both help me remember").

"Do we *have* to do these things?"

My answer to this question is, "You don't *have* to." No matter the theoretical discussions referred to above, or the general agreement in the group to try new ways of learning math, any individual is free to choose whether or not to take part. This is fundamental to my stance as a teacher—a refusal to get into a power struggle with a student about the way learning will take place, and a desire to honour their resistance to being put into the traditional one-down role of the student.

My hope is that this position of choosing will allow them to see the advantages of the activities, so that they might decide it would be worth overcoming feeling silly or awkward or stupid. In any case, if I insist, if I answer this question with a list of reasons why they *have* to, we are in a power struggle where we both lose. When teacher and student are in a power struggle, there is no "teachable moment." The student can win only by refusing to take part in the class, perhaps not coming except to write tests, or perhaps dropping out entirely. I can win by withholding credit, or, in extreme cases, by having him expelled. Both these positions look like winning, and indeed do win the battle, but in fact, they are losing positions as far as teaching/learning goes. The students' right to choose is the only position from which we can both truly win.

Dealing with Student Resistance

Honour it. Welcome it as a sign of students taking responsibility for their learning. Don't get into a power struggle with it.

Evaluate teaching strategies with the class

Before I introduce a teaching strategy that is new to the class, and that I think might meet with resistance, I present it, giving my reasons for thinking it would be valuable. I ask for their reactions, then propose that we try it out for a reasonable length of time, for example, three weeks, and that we evaluate it briefly at the end of the first week, and more thoroughly after the trial period. I make it clear that I will act on the decisions made at this evaluation, and stop using the strategy if most students don't like it or don't find it useful.

If the class agrees at this point to try the strategy out, I explain it in a little more detail, giving an example. I ask the class to predict what effects the new strategy might have. How might it be useful to visual learners? Kinesthetic? Auditory? How might it increase understanding, improve memory, or lead to better test scores? How might it make math class more enjoyable or interesting? All this discussion gives you and the students something to watch for as you begin to use the strategy. At the end of the first week, a brief discussion reminds people what to look for, allows everyone to give an initial response, and reminds everyone that you are trying out something new.

At the end of the trial period, I do a more thorough evaluation. Sometimes I use an evaluation sheet like the one below, and sometimes I ask students to generate a list of questions they would like answered, and turn that into an evaluation sheet, which I administer anonymously and report to the class the next day.

I always keep my part of the bargain—that is, I abide by the evaluation of the class. Why would I force people to do something they believe is not useful and tell me is not useful?

Name of strategy for learning math ___________________________					
Circle the number you agree with. 1 is low, 5 is high.					
	not at all		a little		a lot
This strategy is interesting.	1	2	3	4	5
	not at all		a little		a lot
This strategy is easy to do.	1	2	3	4	5
	not at all		a little		a lot
This strategy is fun.	1	2	3	4	5
	not at all		a little		a lot
This strategy is useful.	1	2	3	4	5
	not at all		a little		a lot
This strategy helps me understand math.	1	2	3	4	5
	not at all		a little		a lot
This strategy helps me remember.	1	2	3	4	5
	not at all		a little		a lot
This strategy helps me talk about math.	1	2	3	4	5

Table 2 A simple tool to evaluate teaching strategies

Figure out who is resisting and who is on side

Often, the students who resist new ways are the loudest or the most problematic in terms of classroom management, while the students who are happy with your new methods are quiet about their feelings. A simple feedback technique is useful to find out how many are with you, and how many are not. With the students, generate on the board a list of the teaching strategies you use, such as group work, work with manipulatives, working at the board, lectures, field trips, tests, assignments, etc. Ask them to list them and consider how useful each strategy is in helping them learn math. They could indicate "very useful," "somewhat useful" or "not useful" for each strategy. Ask them to hand in their papers with no name attached. Calculate the responses for each strategy, and report to the class the next session. You may find that most of the class thinks that a particular strategy is helpful, even though a few protest loud and long when you bring it out.

Knowing I have allies in the class, albeit silent ones, helps me persevere in using a given strategy, in spite of vocal protests. Furthermore, reporting back to the class, so everyone knows where we stand on the issue, helps too. I can be firmer about using the strategies many students find useful, in spite of resistance, and it gives me a chance to acknowledge the feelings of discomfort and resistance I know are there. For example, I might say to the class, "On the feedback sheets yesterday, 2/3 of you said going to the board was useful. So I'll be asking you to go to the board two or three times a week. I'm hoping that those of you who don't like it very much will put up with it because other people find it useful. I know now that many of you are reluctant when I ask you to do it, and I'd like to figure out some ways to make it more comfortable, while continuing to do it. Does anyone have any ideas about how to make it more comfortable?"

Why deal with student resistance?

Student resistance will weigh you down and tire you out. If you don't deal with their resistance, and get them onside with you to work in new ways, their resistance will win. You may give up entirely and go back to teaching in ways you know are ineffective, because that is what students expect you to do.

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"They have a story about math locked in their hearts." — Carolyn Bowles, Faculty of Education, Malaspina University-College

Emotions and Math

"But I'm not a therapist…I'm a math teacher!" I can hear you say it. And you are right. Nonetheless, whether you are a therapist or not, the emotions in your math class are not going away. So how can a math teacher who is not a therapist deal with emotions? First of all, you don't have to be a therapist, just a human, to deal with your own emotions. Do that first.

However, to deal with the emotions in your students, enlist all the help you need. Other people in your program may be very useful in this case—a counselor, or someone who teaches study skills, or does student support, for example. Ask one of them to do a session with your class on any one of the following topics, and, since you're bringing them to the math class, ask them to put a "math spin" on their presentation:

- Positive "self talk"
- Stress management
- Maintaining a positive relationship with your instructor
- Becoming a successful student
- Test anxiety

It's important that you attend the sessions as well. Your presence sends the message that you think the material is important to the math course; that you respect the person giving the presentation; that you want to hear what your students have to say during the session; that you believe there will be a carry-over into the math classes and you want to be part of the carry-over. Your absence sends the opposite message.

Expressing Feelings[1]

An important strand of my teaching philosophy is to deal with emotions, my own and the students', so they don't get in the way of the learning. This comes partly from my training as a Life Skills Coach (Saskatchewan NewStart Model). My coach trainer, Audrey Adilman, used to say that it will probably take less than a minute to acknowledge the emotions that come up in the moment; that if you leave it for an hour, it might take two minutes to deal with them; if you leave it until the next day it might take half an hour, and if you leave it for longer, who knows how long it will take?

How does expressing our feelings help? It helps us keep control of our emotions, helps us identify problems, and helps us maintain clarity in our relationships with other people.

According to the Life Skills lesson on expressing feelings (*Core lessons for life skills programs*, 2004), saying "I'm frustrated" or "I'm mad" or "I'm happy," releases the hold the emotion has on you a little, so that you can concentrate on other things, and think and act rather than just emote. Maintaining control over emotions is helpful in the classroom where so many people are working in a public space. Certainly as an instructor, I'm happy when people can control their emotions so I don't have to control the people. I'd much rather someone say, "I'm really frustrated when I keep getting these questions wrong," instead of slamming his books down and stamping out, swearing under his breath, or out loud. Furthermore, a student who can say what is bothering him may be able to go on working, or ask for help, or use some strategy he has for dealing with stress or anger.

Sometimes a problem shows itself first in a feeling—we notice our muscles are tense, or our palms are sweaty, before we can articulate what is wrong. Paying attention to feelings can help us identify problems: "Every time you mention denominators, the pit of my stomach falls out." "Whenever you lean over to look at my work, I stop breathing." Such feelings help pinpoint a problem, and identifying a problem is the first step in solving it.

Expressing a feeling, and owning it as our own, helps maintain clear relationships with others. If I am worried, but don't say so, the worry may look like anger to someone else—withdrawal, blank stare, drawn-in shoulders, silence, holding my breath. If a student looks at those behaviors and assumes I'm angry, suddenly I have two problems—the thing I was originally worried about and a relationship with a student that needs to be repaired. If I can say, "I'm worried because I'm not sure if we can get this whole activity finished in the time we've got left, and it won't be good to leave it half-done until Monday," then the student is clear that I'm not angry at him, and I have only one problem to deal with—a scheduling problem, not a relationship problem.

If you are uncomfortable dealing with students' emotions, or with encouraging them to express their emotions, keep in mind that the emotions are still there, even if everyone is

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[1] I am indebted to Karen Burns, my colleague at Malaspina University-College, for her help in elaborating the ideas in this chapter.

ignoring them. It may help if you acknowledge possible feelings to the class, without singling out any particular person. For example, you might say, “I can see some of you look puzzled right now. If you are feeling confused, hang on. I think the next activity will help sort things out.” If a student is able to say, “I’m confused right now,” the expression of the feeling releases it, and leaves a calmer space for paying attention to your assurances that help is at hand. If you express out loud your awareness that some people may be confused, you may accomplish the same thing for some students. On the other hand, if you notice the puzzled looks, and go on to a new explanation or to the next activity, without acknowledging the confusion, the students who are dealing with the emotion will not be able to give their full attention to the activity that you hope will clarify the confusion. Expressing the feeling helps the situation, no matter who does the expressing. Students who would be unwilling or unable to express their confusion may also be served by your acknowledgement of it. You will see smiles and nods to confirm your expression.

In the following sections, I’ll talk about how some emotions might be expressed in the class and some ways of dealing with them when they show up.

Joy

I’m happy. I’m having fun. I’m enjoying myself. I’m excited about this. I love math.

I look for joy in my teaching. Why else do it? In teaching math, there is both the pleasure of math to be looked for, and the pleasure of teaching. In learning math, there is the pleasure of math to look for, and the pleasure of learning.

The pleasure of math is a pleasure that both teacher and student can share; the pleasure of finding a pattern, or figuring out three ways to show that 1/8 equals 12.5%, seems to be pretty much equal between me and a student. The student has the joy of discovery, and for me, the pleasure of the patterns does not seem to diminish, and I have the additional satisfaction of witnessing the student’s joy. The pleasure of teaching and the pleasure of learning often happen at the same time—it’s that moment when someone says, “Aha!”

Confidence

I think I can. I’m going to give it a try. I know I can figure this out.

As teachers, we often have the mindset to see where students have gone wrong, to find the errors. This is a useful mindset to the instructor—it helps us figure out what to review, to notice how one way of explaining is more useful than another, to notice patterns of errors, and so on—but it is not useful to the student. The mindset of seeing errors improves teaching and learning when I keep the information to myself. When I say it out loud, it decreases confidence in students who hear it.

It is confidence that allows students to make decisions in math, to decide how to tackle a problem, to believe they can tackle a problem. If they have no confidence, they will take no risks. To believe they can do math in the present or the future, they have to believe they have been able to do it in the past. In encouraging this belief, it is more useful to the student if the instructor is ready to notice and comment on what they have done right, rather than what they have done wrong. Part of that mindset is to cultivate the habit of cutting math problems into small pieces, so that the small parts they did correctly can shine out, and you can acknowledge them.

Students are aware of how you respond to other students. Responding positively to one student may prompt another to take a risk, anticipating that your response will be similar to the one he has witnessed.

Marking in a positive way

Ideally, I would mark each question as the student does it. When students start to work on a page in their books, or on a sheet I have given them, I don't wait for them to finish and hand it in, rather I move around immediately, marking each one as I go, then coming back when they have a few more questions done. When I find several people having the same difficulty, I can call the class together, apologize for not having been clear enough in my explanations, and go over it again.

When I am working one-on-one to mark a student's work, my job is to be encouraging by pointing out what the student has done correctly, and what evidence I see of good math thinking. My job is also to give the student a chance to articulate what she is doing, to help her remember, and to give her control over the process.

When I look at a page with the student, first, I put a checkmark beside every correct answer. I ignore the ones that are wrong, and comment that the student got some/many correct.

If there are only a few wrong, I start with the first one that is correct. I ask the student to tell me how she did it, so she has a chance to "rehearse" the procedure and articulate it clearly. I repeat with the next correct one, and the next, until she can easily articulate the procedure. Then I move to the first one that was wrong, and again ask the student to explain what she did. Usually, she will find her mistake and correct it on her own. I mark it right, and praise her for being able to find her own errors without me pointing them out. I continue with the next one wrong; at some point, I can ask her to independently check all the ones that don't have a checkmark beside them.

If more than a few questions are wrong, I find the first one that is right, mark it right, and ask the student if I can guess what she did. I go over the question, teaching and dialoguing with the student about the method. I try to figure out where the student's error is coming from. I then present a new question to the student and ask her to do it while I watch/coach. Then another. When I am sure the student has the process in mind, I offer a clean copy of the worksheet to do, or a new worksheet with similar problems.

I take my share of the responsibility for asking the student to do something she was not prepared for, so she knows that her mistakes were part of a complex process that involve my explanations, her ability to attend to them, time pressures, her previous knowledge, my knowledge of her math level, and emotional factors.

Fear

I'm scared. I'm anxious. I'm worried. I hate math. I'm stressed out.

Fight, flight, or freeze. I've learned to recognize all these responses by math students, and gone on from there to take it less personally when students attack me or run from me or disengage. I know it's not so much me they are reacting to, but to the situation itself.

For some years I would go around the class, asking, “How are you doing? Do you need any help?” and students would say, “Okay,” or “No.” Usually they kept their work hidden when they answered this way, but often I would find out later that indeed they did need help—they weren’t doing okay at all. Yet they shut me out by saying, “I’m okay.” Why do they lie? Because they are running away from whatever mini-lesson I might give them if they admitted they needed help. Because they are running away from the panic they would feel if they worked on math with the teacher.

Sometimes I would invite people to come to see me outside class time to get some extra help, and the answer might be, “No thanks, I’ll work with my tutor (or my father or my girlfriend or…). But I would hear from the tutor that they didn’t show up for a scheduled tutoring session, and I would see no evidence that the alleged sessions with family members bore any fruit. Why would a student invent math learning at some other time? Because they are running away from my math lesson and from panic.

For a while I took it personally, all this running away, but eventually I learned some tactics for heading it off. I no longer ask, “Do you need any help?” Instead I say, “What question are you working on? What can you tell me about your thinking about that question?” or “You don’t look happy. What’s getting you down?” The student can still avoid me if he wants to, but I don’t make it easy for him. If the student is not struggling, this technique invites the student to articulate their math thinking.

Many students who have been dealing with fear of math for a long time have developed a defensive fall-back position, which expresses itself as “I’m no good at math (so I don’t have to try)” or “I can’t do tests (so don’t ask me to).” Another fallback position is to blame the teacher. More situations for me to practice not taking things personally!

Anxiety

“Anxiety results when you are required to stay in an uncomfortable situation where you believe you have no control,” according to Cheryl Ooten (2003) in *Managing the Mean Math Blues.* How can an instructor help a student take control? See Chapter 7 of this manual, and, in the meantime, since we are talking about emotions, there are some things you can do.

First, acknowledge the anxiety. Talk about the anxieties you have about your own performance in class, for example, first day jitters, worries that you will forget people’s names, or that you will run out of material before you run out of time—whatever they are. When you acknowledge your own anxieties, you disrupt the power imbalance; you become more human. The students have a chance to be generous with you, to *give you* something rather than always being on the receiving end. They can offer to help you remember names, or cut you some slack if your worries come true. (I’m not suggesting that you rely on students to meet your needs for reassurance and support, just saying that an acknowledgement of your own humanity builds a relationship of mutual respect.) Admitting your own anxiety starts the modeling of what you would like the students to do—if the teacher can admit she is anxious about goofing up, then maybe the anxious student can do so, too. If that happens, you have some information about your students and their areas of need.

Strategies

Talk about the way stress shows itself in the body. Strategize with the students some ways to deal with stress. They will have some good suggestions, especially if they have had experience in life skills, anger management, or Alcoholics Anonymous. Ask someone to come into the class to teach a few methods of dealing with stress so that students can choose something that works for them; learn them yourself, and model them by using them with the students. Share your own methods. (Mine is square breathing.)

Square Breathing

Get ready to start square breathing by taking a deep breath and releasing it. There are four steps in each set: inhale, hold, exhale, and hold. Give an equal length of time to each step. Concentrate on counting and breathing. Count four for each step:

- Breathe in, two, three, four
- Hold. two, three, four
- Breathe out, two, three, four
- Hold, two, three, four

Repeat several times.

Above all, make it clear that you expect, welcome, and allow students to use these methods in class. For example, if one strategy is to walk around the block, you will not call attention to someone who gets up in the middle of class to do that—and you will be happy to see him return to class 10 minutes later. Such stress-reducing activities as leaving the room or walking over to the window to look at the trees or the sky are not customary in classrooms, and students will need to be reassured that you mean it when you ask them to use some strategies for relieving stress, strategies which often require movement of some kind.

Test Anxiety

I understand it in class, and I get my homework right, and on the test I freeze.
I forgot my name. My blood froze. I could feel it running through my veins with little ice cubes in it. I could feel the little ice cubes bumping along.
I remembered how to do it as soon as I walked out of the test room.

What can a teacher do to reduce test anxiety? Here are some suggestions:

No surprises: Make a deal with your students that there will be no surprises on the test. If you use a pretest, it will look like the real test; if you say what kind of questions will be on the test, those questions will be there, in the shape and form they expect. There will be no surprises with the language. If you use the term "reduce" in class, then the test will say "Reduce these fractions" not "Write in lowest terms." If you ask in class, "What is the value of x?" the test will say just that, not "Evaluate."

Teach test strategies: Ask someone to come to class to give a session on studying for and writing tests, or give a session yourself. One strategy I learned in such a session is that when you are studying for a test, you do what the test will ask you to do. So when studying for a math test, you do examples of the kinds of problems that will be on the test. You don't study for a math test by reading your text or watching someone else do math, because the test will not ask you to read about math or watch someone do it. The test will ask you to do math so you study by doing math!

Encourage students to ask questions during the test: This practice has saved me many times, after I've let a typo get through without noticing it, or a dirty photocopier has put in a decimal that I didn't mean to be there. After the first student notices my mistake and asks me about it, I can correct it for everybody. Of course, I don't want to help people do the math on a test—that is their job. However, I want them to be able to show what math they can do, and if they don't understand my instructions, or find an impossible question, I want to clear up the misunderstanding so they can go ahead and do the math.

Offer students a chance to do the test privately: Karen Burns reports that at such private test sessions, she begins by reading each question to the student, and then writes down what the student tells her to write. Later she reads the question and the student takes over the writing; she lets this happen on the student's schedule. Soon, she says, the student says s/he no longer needs to do tests privately. In her experience, most students say they want to take tests in the usual way after only one or two private writings, although one student needed to take four tests privately in this manner.

Boredom

I'm bored. I'm tired of this. I feel blah!

I have a rule in my class, that I follow myself: Refuse to be bored. I find it very useful to encourage students to say when they are bored, and to refuse to be bored in math class. The rule introduces something new. Generally, they expect to be bored in math class; for many, a mix of fear and boredom is exactly their experience of math class. So, when I ask them to commit to saying it out loud when they are bored, and suggest that math class should be exciting, or at least interesting, I have changed their expectations.

What happens after the student says he's bored? Again, the way the teacher takes this feedback is important. Sometimes the temptation to retaliate is great. When someone says it's too easy, the temptation is to leap up six levels and give him something really hard! After all, he messed up your lesson plan, so you want to mess up his. If I can take this feedback less personally, I can begin a conversation about what makes the material boring. A bored student is a disengaged student, but a student who says he is bored gives you an entryway into dealing with some math difficulties, and into re-engaging the student. I try to find out if the student is bored because it is too hard. (I know from personal experience that frustration gets boring very quickly.) Is she bored because it is too easy? Because he doesn't see the point of this particular thing? Because they've done it all before, and nothing seems different this time around? Whatever the reason, it is sometimes easier for a student to admit to being bored than to admit to being scared or frustrated.

Anger

I'm frustrated; I'm irritated; I'm mad.

I like to say when I'm irritated or frustrated, or mad, for two reasons. First, saying it helps me calm down a little. Second, it gives me a chance to make it clear what I'm frustrated about. Even if I don't say anything, the students know I'm irritated about something—the emotion leaks out in my voice or my body language, and they are past masters at reading voice and body language of teachers. They often assume the worst—

that I'm mad at them because they are stupid. Saying what I'm frustrated about is useful to them as well.

For example, I might say, "I'm feeling frustrated with myself. You and I have been working really hard on this, and I can't seem to find an example or an explanation that will help. My brain seems to have frozen solid. Can you let me think about it overnight, and maybe I can come up with something that will be useful to both of us." (Here I make it clear that I'm frustrated *with myself*, not with the student. The student gets to see that I'm human, that teaching is work, and also gets a chance to be generous with me by giving me some time to come up with something new.)

or:

"I'm frustrated when you miss so many classes. It's hard for me to help you catch up, and I worry that even if you do catch up today, you might not come tomorrow, and then the next day we'll be back at square one. That makes things hard for you, and it makes my job harder too." (Here it is clear that the attendance is the problem, not that the student is stupid. It seems to me that it is easier to come to some solution about lack of attendance than to cure stupidity. It also makes it clear that his absences have an effect on me, that I want him to attend regularly not "for his own good," but because it makes my job easier and more fulfilling.)

Students also get angry in math class, and often they direct their anger at the instructor. "You don't like brown people." "You only work with the pretty girls." "You don't care about people who've been out of school a long time." "You don't understand youth." "You don't…" Students lash out, and it hurts, and your own emotion interferes with your ability to teach that moment or that student. Once again, this is a place to acknowledge my feelings, figure out where the student's anger is coming from, and go on from there.

Humiliation

I feel like I'm in kindergarten.

Often students haven't done many participatory activities since they were in early elementary school. I hear many reports that my students were so far behind in school that they didn't work with the rest of the class, rather worked alone in a workbook that the teacher marked and gave back to them. So when I ask them to use manipulatives, for example, I know they sometimes start by feeling humiliated, because in their experience, manipulatives are for babies.

I know that a student whose mind and heart is occupied with feeling humiliated and resentful about using the manipulatives will not see that 2/6 = 1/3, and since my interest is in exactly that, I'll do something to help express the feelings, so we can both get on with the job we came to do. I am happy to hear people express their own feelings without prompting, but if a student is not expressing anything, just sitting with hands in pocket, I may make a stab at identifying the emotions. I might say, "When I first started using these I felt clumsy with all these little pieces," or "Sometimes students tell me that the blocks are just for kids, and they feel silly using them." That is often enough to open the way to the student expressing the feelings, and once that is done, a rational decision can be made about if and how the manipulatives will be used.

For those who like manipulatives, it is play, and I feel child-like, not childish, when I use them. So I like to let my enjoyment of manipulative play show, and encourage students to have fun, too, but I am careful to call the manipulatives themselves “math tools” and not “math toys,” which is how I think of them privately.

Trauma and Learning Math

I’m not allowed to have opinions. I don’t have any opinions worth expressing.

I know from my reading, [for example, *Too Scared to Learn* (Horsman, 1999), and *Violence and Learning: Taking Action* (Norton, 2004)], and from my experience, that students who have experienced violence may space out or act out in class, no matter how motivated they are to learn math. Particularly I notice how difficult it is for many of them to share an opinion or make a decision. Expressing an opinion has been dangerous in the past, and it is not possible to do math while avoiding expressing an opinion about how to solve a problem. Making the classroom a safe place to take risks is an art. When you involve the whole class in figuring out how to make it safe for everyone, you show your commitment to safety and offer a chance for students to say what they need to be safe.

Making the classroom safe for the many students who are survivors of trauma is beyond the scope of this manual. Jenny Horsman, Mary Norton and others have written extensively about the relationship between violence and learning, and much of their material is on line at www.nald.ca. An exciting new website being developed by Jenny Horsman is at www.learningandviolence.net.

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I really do believe that if students in high school were cutting pieces of cheap plastic pipe, they could carry away a better picture of trig. Two years later they won't remember everything but they will have a more concrete memory of what it is and how it works. — Sue Grecki in "Making math concrete (and iron, and plastic): Numeracy and construction trades" (2005)

Hands-on Learning

When we teach any science, we recognize the importance of lab work, yet often math is taught only in the abstract. However, in real life, math is nearly always hands-on: we count the money, double the recipe, measure the wood, or estimate how to make the treats last until the final kid has come to the door on Hallowe'en.

Manipulatives

As I was talking to instructors in preparation for writing this manual, they told me that they found many barriers to using manipulatives. Some said their programs didn't have any, and there were no funds to buy any; some instructors themselves weren't comfortable using them, so they weren't comfortable using them with students; some were afraid that the students using manipulatives might get into places that the instructors had no explanation for; many said that their students resisted using them. These are major barriers, and the results of using manipulatives had better be worth the time and mental energy it takes to overcome them.

Why Use Manipulatives?

I think most people would agree that manipulatives such as base ten blocks or fraction pieces provide a model of mathematical operations to supplement verbal explanations (themselves abstract) of abstract processes. For example, the two cubes that show 1 and 1 000 are clearly not the same size, and illustrates physically the difference in the value of the digit 1 in the numbers 1 and 1 000.

Manipulatives slow down the action
Going further, however, I find that using manipulatives slows down the process of explanation so students have more time for understanding. For example, in showing 24 – 19 using blocks, the time required to exchange 1 of the tens of the 24 for 10 ones is much longer than the time required to say "Regroup the 24 into 1 ten and 14 ones" or "Borrow 10 from the 20 and write 14 in the ones column." This extra time gives the student a moment to absorb what is happening before you go on to the next step.

The student controls the pace of the work
Furthermore, if the student is using the manipulatives himself, and not just watching, the student controls the pace of the work. Students who are not sure of themselves move more slowly. If you watch students working with manipulatives, you get a sense of their understanding. You can't see what's in their heads, and often they cannot articulate their thoughts, but you can tell by their hands where their understanding is faulty.

Students get just as frustrated as anyone does when the manipulatives take more time than is necessary, and that spurs them on to making up shortcuts. To continue with the example above, when doing subtracting problems with the blocks, eventually students get tired of counting out 10 ones in order to exchange them for a ten-rod. They may set aside a little group of 10 ones instead of putting them back in the general pile, so they don't have to keep counting them out again and again; they may start to count off the pockets on the back of the ten-rod instead of dealing with the loose pieces, or they may invent some other short cut. All of these are indications of a growing understanding of the operation of subtraction, and of a movement towards abstraction, which will allow students to do the operations using only pencil and paper, or in their heads.

Yet the word "necessary" in the first sentence of the previous paragraph is key. When the student is controlling the pace, she will continue to use the full process as long as it is necessary for her understanding. When she has not fully grasped the fact that subtracting 19 from 24 requires 1 of the tens in the 24 to be changed to 10 ones, counting out the 10 ones, exchanging them for 1 ten from the big stash, checking that she now has enough ones to take 9 ones away, then setting them aside and counting what is left—all these are absorbing and necessary tasks, and do not seem to take up "unnecessary" time. As she begins, through repetition, to become familiar with the process, as her understanding increases, her impatience with the physical process increases, because it is no longer a necessary piece of the action of subtracting. Her impatience will lead her to a shortcut, and eventually to doing the question without using the manipulatives.

Manipulatives help students remember
The use of manipulatives provides memory cues for different kinds of learning. The movement of the arm in operations of addition or subtraction, multiplying or dividing, reinforces the meaning of the operations for kinesthetic learners. In the operation of addition, the arm sweeps two or more groups of blocks together. In subtraction, the arm

moves to separate a part from the whole. In multiplication, the arm moves repeatedly to add a group a particular number of times. In division, the arm makes the sharing-out movement that card players or parents of small children recognize immediately. The shapes, colours and sizes of the manipulatives provide cues for visual learners, and they carry those images with them when they move to working mentally or on paper. Finally, since talking seems to go with using manipulatives, auditory learners get to tell themselves stories about what they are doing, and hear others talk about the processes they are demonstrating, and this verbal rehearsal of the process is committed to memory.

Students get the right answer

Most important, the students nearly always get the right answer when they use manipulatives. If you ask a student to add 1/3 + 1/6 using manipulatives, the answer will never come out to 2/9, which is a common error students make when adding fractions on paper. This means that the instructor deals with a student who has the correct answer. Rather than having to deal with an error, you can work on extending understanding, or helping the student articulate the concepts. The benefits in terms of student self-confidence are evident.

Finding, Making and Buying Manipulatives

Commercial manipulatives

The advantage of commercial manipulatives is that they are accurate and available. Plastic pieces made in a factory are precise—they will stack as needed, and 2/10 are exactly equal to 1/5; 10 tens are exactly equal to 100. They cost money, not teacher time, and they are ready for use. If you live in an urban area, there may be a teachers' store that carries a variety of manipulatives and models. Outside of major centres, if you want to try before you buy, check with someone in the local K-12 system to see what they can show you. Anyone can look on-line for what is available; here are some places to start:

Artel Educational Resources, in Vancouver, sells many different types of manipulatives: http://www.arteleducational.ca/index.php

National Library of Virtual Manipulatives lets you try out, virtually, many different kinds of manipulatives that are available in real form for students:
http://nlvm.usu.edu/en/nav/vlibrary.html

Center School District lets you cut strips into fractions, decimals, and percents. Fun for you to play with. Scroll down for instructions.

http://arcytech.org/java/fractions/fractions.html

Home-made manipulatives

Teachers can make manipulatives that are reasonably accurate and inexpensive, but it takes time. Some suggestions are in the sections that follow. Making manipulatives with students is a good experience, but the results are sometimes not accurate enough for precise comparisons. As a process, it is social, provides lots of time for understanding, and provides lots of repetition with a purpose. It means that every student has a set, and knows how to make more for children and family.

Manipulatives for whole numbers and decimals

Student activities using these manipulatives start on page 52.

Place value blocks

These blocks are easier to use and put away if you buy a big plastic bin to hold them, and a small plastic box to hold the little ones. They come with a couple of sheets of activities for exploring the set.

Extend the set: If you have these blocks, it is interesting to work with students to extend the set so you can show very large numbers. A stack of ten 1000 cubes will give you an idea of the size and shape of 10 000. For each 10 000 rod, you will need a carton about 10 cm by 10 cm by 1 m; 10 of these will make a 100 000 flat. For this flat, 100 000, you will need a carton about 10 cm by 1 metre by 1 metre (the shape of large, square floor fan). 10 of these piled up will show 1 000 000. A million cube needs a carton about 1 metre by 1 metre by 1 metre. You will probably have to go outside to pile up 10 million, unless you have a high-ceilinged foyer somewhere.

Make your own: Getting students involved in making their own manipulatives is easy to do. For example, use toothpicks for a value of 1, put an elastic around 10 toothpicks to show 10, put 10 of these bundles in a baggie to show 100, and 10 baggies in a big envelope to show 1000, 10 envelopes in a bulldog clip to show 10 000. When your students get tired of bundling toothpicks to fill the baggies, they will begin to use empty baggies as symbols of 100. That's the day you can celebrate their understanding and their increasing comfort with the abstract!

Make a collection

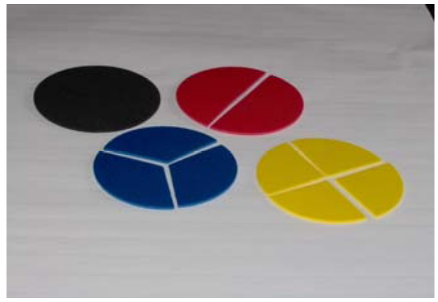
Making a collection of a million things is a very interesting procedure. Sometimes you see reports of classes collecting 1 000 000 tags from packages of bread. Collecting a million pennies would raise $10 000 for your program. In any case, making a collection of a million things usually involves publicity, since the wider community is needed to help make the collection, which means that students can talk about the process of showing what a million is; they are involved in making a system for storing, sorting, and displaying the pieces they are collecting; it takes time, so the immensity of the number has time to set in, and it provides a repetition of noticing smaller amounts—10 tens make 100 and 10 hundreds make 1000 many, many times during the process of collecting a million of anything.

There is lots of room for ratio and proportion in this project: if 100 items weigh five grams, how much will 1 000 weigh? 10 000? 100 000? If it takes three minutes to count 100 pennies and put them in the plastic case, how long will it take to deal with 1 000 pennies? How long will it take to wrap the million pennies?

Manipulatives to teach fractions

Student activities using fraction manipulatives start on page 72.

Commercial manipulatives



Some examples are shown above. See page 26 for sources of commercial manipulatives.

Make your own

Patterns for fraction strips: Photocopy each of the following pages, making sure your photocopier is set to copy exactly 100%, not 1.1 as is often the case. I colour code the denominators of the fractions by using a different colour of paper for each pattern page.

Page A: Use for halves, fourths, eighths, sixteenths. Cut the pages lengthwise along the solid lines. Students can fold the strips in half crosswise, fold in half again to show fourths and in half again to show eighths. Another fold will make sixteenths, but the thickness of the paper makes this fold difficult; an alternative is to open the strip up again and fold in half lengthwise to show sixteenths. Fold marks indicate the division lines, so there is no need to mark them in with pen unless you want to.

Page B: Use for thirds, sixths, and twelfths. Cut the pages lengthwise along the solid lines. Students can fold the strips along the dotted lines to make thirds. Once folded, they can fold in half to show sixths, and in half again to show twelfths. This final fold can be lengthwise or crosswise. Fold marks indicate the division lines, so there is no need to mark them in with pen unless you want to.

Page C: Use for fifths and tenths. Cut the pages lengthwise along the solid lines. Students can fold the strips along the dotted lines to make fifths. Once folded, they can fold in half to make tenths. This final fold can be lengthwise or crosswise. Fold marks indicate the division lines, so there is no need to mark them in with pen unless you want to.

A

B

C

Egg Carton Fractions[2]***:*** With empty egg cartons you can demonstrate equivalent fractions, proper and improper fractions, and addition, subtraction, multiplication and division of fractions using a whole dozen, half a dozen, thirds, fourths, sixths, and twelfths of a dozen. One advantage of egg carton fractions is that students can make them themselves and inaccuracies of cutting do not render them useless, since it is the number of cups that makes the fraction, not the precise size of the cut.

Two ways to show 1/3 of a dozen

Further, it is clear that six egg cups is half a dozen, whether the six are in one long row, or two rows of three. Other advantages are their cheapness and their stackability!

Two ways to show 1/4 of a dozen

To make a set of egg carton manipulatives, you will need six empty egg cartons. Start by taking the lids off them all. Then work on the bottoms. Leave the first carton whole (12 egg cups). Cut the next in half (6 cups in each half). Cut the next into fourths (3 egg cups in each quarter), the next into thirds (4 egg cups in each third), the next into sixths (2 egg cups in each sixth) and the final one into twelfths (1 egg cup in each twelfth). Students can stack various pieces inside each other to show that 6/12 equals 1/2, that 3/4 equals 9/12 and hence is larger than 2/3, which equals 8/12, and so on. You can ask students to do the fraction demonstrations starting on page 72, using these egg carton manipulatives, but they will be able to demonstrate only questions with denominators of 2, 3, 4, 6 and 12.

Charts, Diagrams and Other Visual Representations

It is important that students make charts and diagrams themselves, and use their own creations as tools for doing more abstract calculations. The temptation is to simply hand out copies, but, in this case also, making their own visuals slows down the pace of presenting the concepts, puts the student in control of the pace, and builds associations in the student's mind as the visual is being made.

Even something as simple as a times table chart will have more meaning if students make their own, rather than you handing it to them.

Showing Information on Charts and Graphs

The best way for students to learn to read graphs and pie charts is to make many of them. Learning to make charts or bar graphs or line graphs can be taught in math class,

[2] This idea is from Cheryl Ooten's book, *Managing the Mean Math Blues* (2003).

but they can be assigned in other classes as well. You may have software available that students can use, or go to this web site that allows students to make all sorts of graphs and charts: http://nces.ed.gov/nceskids/createagraph/

I like to assign a graph or chart every day for a month or longer, starting with bar graphs to compare two (or more) things, then a line graph to show how something has changed over time, and then a circle graph to show how a whole thing is broken into parts. I provide some data, ask for the essentials (below) and students decide on what labels to use, what colours, what type of chart. I ask them to post their finished charts, and we take a look at what makes a chart readable. I don't point out any errors, or make any critical comments. I comment on what makes various charts great. For example, I might note that the bold colours on Mary's chart make it easy to see the difference between the columns, or that the clear title on Pete's makes it easy to understand what the graph is showing, or that the large scale of Mustafa's makes it easy to take everything in.

What are the essentials?

- A title that says what information is being presented
- On bar graphs, both axes need to be labeled with words, and one with numbers.
- On line graphs, both axes need to be labeled with words and numbers.
- On pie charts or circle graphs, each part of the circle needs to be labeled with words and a percent.

Students will do many graphs, so it doesn't matter much whether their first ones are perfect—they will refine them as they go along. Point out the parts they have done well, be specific about what makes those things good, and ignore the rest. The next day they can try again, and will feel confident about their ability to do it.

I don't worry if they haven't done percent yet when I start graphs. They can learn the single operation necessary—find x% of 360°—and practice it over and over, and it will be familiar when they get to the section on percent.

5

Fear of exposing ignorance seems to be the motivator for these folks to want to do their own individual work. — Iris Strong, ABE math instructor

Group Work

An interesting discussion with my colleagues led to an illuminating discovery about students' reactions to doing group work. Some teachers were talking about how hard it was to get students to do group work: they drag their feet, complain, and say they'd rather work on their own in their books. Iris Strong, who teaches ABE at the Nanaimo Campus of Malaspina University-College, speculated that her students thought working in groups was inefficient and time-consuming. She thought that they wanted to work on their own in order to make faster progress. The following week, she led discussions in two different classes, and reported that her hypothesis was wrong:

> Then I asked them to describe what happens inside their heads and hearts when they're asked to participate in an activity that doesn't feel natural to them, or one that they simply do not LIKE doing.
>
> Well, I was wrong about thinking it was a time issue! At least for the students who responded in these two classes, the common denominator was that taking part in some of these activities involves interaction, and during interaction, others can see that they are dumb, stupid, and not as smart. Fear of exposing ignorance seems to be the motivator for these folks to want to do their own individual work. The cool thing is that they were all quite comfortable sharing that, and laughed about acknowledging it to each other. We've had a couple of wonderful sharing times on this topic in each class!! (e-mail, February 23, 2006).

We're back to dealing with resistance again, and with emotions, and with making a safe space for students to take risks.

A recipe for group work developed by some members of the Math Articulation Committee called for both instructor and student readiness, but the two largest ingredients were commitment on the part of the instructor and appropriate activities.

To me, commitment means that I am committed to keep on keeping on, no matter what the results are on the first or second day I try it. It means that I have prepared the students for something new and different, and that I have enlisted their agreement to try it out for a couple of weeks and then evaluate it together. It means that I am expecting that it might take a few sessions before we, instructor and students, are comfortable with the format of the activity, and I am expecting that I will have to tweak it a little as I go along.

What is an appropriate activity? There are two aspects to take into account here, the kind of group skills required of students and the difficulty of the math content. When students have strong group skills, when you and they have established a safe atmosphere for taking risks in learning math, then they can, in pairs or small groups, take on some investigations of math beyond their comfort zone, that is, math that is truly new to them, not simply a confirmation of what they already know or have experience with. Such group work requires students to be able to show leadership in a group, to keep a group on task, to disagree without devastating conflict, and to compromise for the sake of accomplishing a task. All of these skills need to be learned, and, in my experience, it is rare to find a random group of math students who come equipped with them.

For the classic group activity—give a group of three or four students a new kind of math problem to solve together and make a presentation on their joint solution—I'd wait until my class had progressed to the point where they were confident about their math skills and liked to solve problems. I'd wait until they had worked in small groups enough to develop good communication skills and to learn each other's strengths and weaknesses. I'd wait until they wanted to get into a situation where they had independence of action. (If other teachers were also interested in working with these same students in groups, we might get together to teach some of those group skills.)

Before they reach this rarefied state, I like to introduce them to group work by doing math activities that are social. Social activities provide opportunities to share math knowledge and experience, and to talk about math, without requiring students to have a highly developed ability to articulate concepts, negotiate meaning, disagree diplomatically, or provide leadership to keep the group on task.

When Group Skills Are Low

Social Activities

In many of the activities that follow, the teacher is leading the group and relatively few group interaction skills are required of the students, so they are good activities to use if you have not asked your students to do group work before. Furthermore, because the teacher is leading the activities, it is relatively easy to adjust the level of difficulty of the math as you go along: you can make it more difficult when the students have understood the concept, or stop the process for a moment to teach something everyone seems to need; you can plan for some whole class or one-to-one teaching before the next group session, to clear up some misunderstandings or pave the way for what you want to practice when you have them all working together again.

Working with Manipulatives

Working with manipulatives becomes social if everyone is doing it at the same time. Students can work on their own individual work sheets, but sit together at the same table, perhaps sharing manipulatives. They can watch each other, talk with each other, share opinions or ideas as they like, but you do not ask them to come up with a joint solution, or to decide whose solution is “correct”; both of those scenarios require group skills of compromise, clarification, ability to disagree, cooperation and so on.

Think-Pair-Share

Another example of group work that does not require high levels of group skills is a think-pair-share activity. Here you present a problem to the whole group, ask them to think about it and come up with a solution individually, then pair up with another student to share their solutions and how they arrived at them. If the problem is one that has a numerical solution, students will be more likely to be interested in each others’ solutions if they have arrived at the same answer, so you might ask them to pair up with someone who got the same answer as they did. Students simply tell each other about their process, and are not required to come to a common solution, or to present a joint analysis. Again, you are interested in the fact that they share solutions, and that they talk together about a math process. Some will be more skilled than others; for many students, their ability to talk about math will be much lower than their skills in computation or problem solving. Give them lots of practice in talking, and accept any kind of talking about math as a step in the right direction. Watch their ability grow with practice.

Get the Group Moving

Any activity that gives people a chance to move around has many benefits: it provides an opportunity for laughter and socializing, which helps make the group safe for people to take risks in their learning; it reduces boredom; it gives a chance for students to help each other unobtrusively; it gives a chance to talk about math, and it is useful for many students, including some trauma survivors, to be able to show the “right” answer without having to speak or assert their opinion, where they can move in a group and so be more hidden. Particularly for students who are kinesthetic learners, movement promotes learning and retention.

Following are a few activities which take only a few minutes to do. Enough examples are given that the activity can be repeated over many days, each time with one or two examples.

Human digits
Ask each person to write any digit on a sheet of paper, large and dark, so it can be seen from across the room (or prepare sheets for the class). Divide the class into pairs, threes or fours or fives, or larger, depending on the size of the numbers you want to practice. Each student becomes the digit she holds. Call out criteria for the numbers you’d like, for example, “Show the largest number you can with your digits.” Allow a minute for the groups to organize themselves, and then give them the signal to stand or turn so the whole class can see. Ask someone from another group to read the number and decide if it meets the criteria. With the whole class you can look at all the numbers, asking who

has the largest, and smallest, or asking the groups to line up in order from largest to smallest. Here are some suggestions for the questions:

Whole numbers: make the largest possible number; the smallest possible number; a number with the largest digit in the tens place (or hundreds place, etc.)

Decimal numbers: Give each group a token (a hackysack, or a chair or a book bag) to use as a decimal and ask the same kinds of questions as above—the largest number possible, the smallest possible number, a number with the smallest digit in the tenths place, a mixed number, etc.

Variation: If you are working with only one or two students, give them several digits and ask them to line up the digits on a ledge or table.

Human fractions

You are going to ask the class to sort themselves into groups of any size that meet the criteria you call out, for example, "groups in which half the people are wearing glasses." Start by saying that there will nearly always be a group that doesn't fit the criteria you call out, and that figuring out that you can't make a group is an important part of knowing math, and that you will be asking that group to name a fraction that describes their group. Then ask the class to get into groups or pairs in which half the people are wearing glasses.

In this example, when students have finished getting into groups where half the people are wearing glasses, you might find several pairs of people. In each pair, one student is wearing glasses, the other is not. You might find a group of four with two people wearing glasses, or even a group of six with three people wearing glasses. And you will likely have an individual or a group of people left over, either all with glasses, or all without. You might have a group of three who want to stick together, with one wearing glasses and the other two with no glasses.

Start with the group that doesn't fit the criteria: What fraction of the people in this group is wearing glasses? Ask them to be your assistants in checking out the rest of the activity.

Check all the other groups: "What fraction of the people in this group are wearing glasses?" (Your student assistants can ask the question, and count to check that the group mix is correct.)

Then start joining up the groups that meet the criteria. Take two pairs, each with half wearing glasses. Join them together as one group, and ask, "What fraction of the people in this group are wearing glasses?" The answer will be 2/4 or 1/2. An assistant can count and check, and another should make a note on the board that 2/4 = 1/2. You are showing equivalent fractions here. Ask another group to join the big one, and ask again, "What fraction of the people in this group are wearing glasses?" Assistants again count and check, and make a record of the fractions on the board. Keep joining groups until all the groups are together. Ask, "What fraction of the people in this group are wearing glasses?" When all the groups have been joined, ask everyone to sit down, and ask your assistants to review the equivalent fractions written on the board.

Other suggestions for forming groups: Ask students to get into groups where 1/3 of the people are wearing watches; where 2/3 of the people are wearing sneakers; where 1/4 of the people are wearing shorts; where 4/5 of the people are wearing pants; where 3/4 of the people are wearing some piece of black clothing; and finally, to find many equivalents for 1, ask for groups of any size where all of the people are students.

Find your match

Print some of the charts starting on page 92 and cut out the notation strips that go with them, which show equivalent fraction, decimal and percent notation. You will need one chart for every three people in your class. Set the charts up around the room. To begin with, for example, set up charts showing 50%, 25%, 100%, 75%, and 10%, enough for 15 people. Shuffle the strips that go with these charts, and ask each student to take a strip and go to stand beside the chart that matches his strip. Once there, consult with the other students he finds there, to be sure they are all in the right place. For example, the three students holding 3/4, .75, and 75% should be standing beside the chart that has 75 of the small squares coloured in. When all the students are standing beside a chart, ask each group to name the equivalencies they show, and explain their thinking.

When your students are new to percent, use charts and strips that will be easy for them. Later, you can use a group of charts and their accompanying strips that show 5%, 5.5%, 50%, 55% and 55.5%, for example, and only later, when students have a good understanding, would you mix 50%, 5%, .5%, and .05%.

The final chart in the series is blank. Make a copy for each student, ask them to shade in some of the small squares, and to label the strips. Use those charts and strips to do similar activities.

Variation: Give out only two strips for each chart; for example, give only the fraction and the decimal notation of the amount shown on the chart. The two students who meet there with the equivalent strips will collaborate on writing the missing third notation (in this example, the % notation).

Variation: A single student or a small group of students can use these charts and strips at a table, and work on matching the strips to the appropriate chart.

Songs

Velma McKay from the College of the Rockies, Cranbrook, BC, finds that teaching her students silly math songs helps keep them positive about math and helps them remember various facts and algorithms. You will find some of her songs beginning on page 89.

Measurement

Almost anything in the environment can be measured, and anything that can be measured can be estimated.

Linear measures

Ask students, working in pairs, to find two or three things they estimate to be about the same length, measure them and fill in a chart like the one below. For example, the length of a table and the height of a door, or the lengths of a particular cell phone, pen, and notebook.

Item	Estimate (with unit)	Measurement (with unit)

Weight

- If you have access to a scale, a similar chart to the one above will ask students to estimate and measure the weight of various items.
- Give students a variety of cans or packages with no labels, and ask them to line them up in order from lightest to heaviest, by feel. Then ask them to check their estimates by weighing the objects. The objects should include things that are big and light, big and heavy, small and light, small and heavy, and the containers should be of various shapes.

Volume

- Collect a variety of mugs and cups, and ask students to arrange them in order from smallest to biggest, just by looking at them, then to check their estimate by filling each mug with water and measuring the water in a measuring cup or a graduated cylinder from the lab.
- Repeat, using various bottles and cans.
- Ask students to look at the coffee pot. When it is filled to the 10-cup level, what size cups is it talking about?

Board Work

Asking students to work together at the chalkboard is a group activity that does not have the problems sometimes associated with doing group activities; when everyone goes to the board, the leadership of the group remains with the teacher, who will perform most of the facilitation needed.

It is a social way of doing math, but each student does his/her own work, comparing it to other students' work, but not having to challenge them about it. It encourages students to look at the work of other students and to ask questions, but there is no onus on the group to come to an agreement and present one response to a question.

The teacher can see everyone's work at once, without having to go around from group to group or individual to individual; it is easy to see immediately who is having trouble and who is not.

Working at the board is easier than working on paper for students, because it is easier to erase a wrong answer and correct it, and students get an immediate response as to the correctness or appropriateness of their answer either from the teacher or by checking with their neighbours. I ask them to take responsibility for making themselves comfortable at the board by finding a long piece of chalk, making sure that an eraser is near at hand, standing beside someone they feel comfortable with, and making sure they can see the work of someone who they think is good at math.

You will find a set of daily activities at the board practicing place value on page 62, and another set practicing operations with fractions on page 83.

More….

Many of the activities in the next chapter, "Real Life Problems," ask students to work in groups or pairs.

6

In talking as though we ([instructors] and policy-makers) know or can predict students' everyday lives and maths problems, we risk being profoundly patronising. What is real, everyday or relevant, or has meaning, depends on things far more complicated than [we] can know. — Alison Tomlin in "Real Life in Everyday and Academic Maths (2002)

Real Life Problems

The strategy, as Ginsburg and Gal (1996) put it, is to "situate problem-solving tasks within familiar, meaningful, realistic contexts in order to facilitate transfer of learning," and a glance at most workbooks or textbooks for math instruction shows how widely this principle is accepted.

Barriers to Using Real Life Problems

This seems so easy. What makes it hard to do? I will suggest some reasons and discuss each in turn.

Students Know, Teachers Don't

As an instructor, I know I don't understand some contexts that learners understand, and so I'm unwilling to work with or make up problems in these areas. I'm afraid I won't be able to get the "right" answer, or that someone will ask me a question I can't answer. I don't trust my students who know about an area to be able to explain it to me or to other students who don't know about it, so I like to be prepared with a back-up explanation. If the area is something that I am not familiar with, I feel lost and unwilling to take the risk. An example for me is sports statistics. I know there are acres of math in there somewhere—batting averages, win/loss ratios, salary caps, comparison of scoring records from former days with scoring stats from today, but it's not *my* life, so I don't feel comfortable working with it in math class.

For me to use this area of real life, I have to learn something new, maybe a lot of new things; if I decide to learn from my students, it's going to be messy, with a lot of "not math" going on in the math class. Maybe I can learn some basics from a book or from a friend, outside of class, and then I'll have to trust my students to help me through the hard parts. There's that shared power structure again.

Teachers and Students Understand Differently

I get fooled sometimes. I'm talking with students about something, and I make all kinds of assumptions based on how it works in my life, but as we talk, I begin to see that things work differently in their lives. Of course, this happens with friends and colleagues, too, but in the classroom I want to find contexts that we share, so that the math part stands out. When I start from my life to find examples of math use, I often find examples that do not speak to my students' lives. Cooking comes to mind. For me, cooking involves reading cookbooks, deciding on a recipe, checking the cupboard, and buying what I need for the recipe. I measure nearly everything with cups and spoons, so it's easy for me to cut a recipe in half, or double it. When I talk to students, however, or watch them cook, I find most don't use a recipe, and they measure by eye or by feel, not by cup or by teaspoon. When I look at the way they cook, I can't find any math; when they look at the way I cook, they don't recognize it as real life, so math work I make up about cooking is not real life math for them. They have to understand this foreign way of operating in the kitchen before they can attempt the math problems, and the result is that I make it harder for them, not easier.

Math Problems Include Hidden Motives

Many texts use problems taken from real life that I think have a hidden message: "Your life is a mess because you don't know how to manage your money and I'm going to show you how." There follow endless questions about balancing a checkbook, finding the best buy, figuring out interest payments, working on commission. For students living below the poverty line, the problem is not how to budget, but how to get more money, food or other necessities. Finding out how to get an extra bag of food from the food bank, or where to go for a free dinner, or how to get some help to transport the free couch someone has offered you, are all much more efficient ways to improve your life than learning to balance a chequebook.

A participant in an on-line discussion group posted this story:

> I was contracted to teach the literacy portion of an employability skills course (students were mandated to take this course through social services) and I was mandated to teach a section on budgeting. I walked into the class, and began talking about coupons, saving, budgeting, etc.—all to blank faces. I could feel the anger in the room. I went home, pondered, and then the next day, I sat myself in the back of the room, (trusting my gut instinct on this one) and said, "Who am I to talk to you about budgeting?" (I knew they envisioned me as having a good income.) I said it was them who could teach me how to budget since they did it every day of their lives—them who had to scrape by on a measly $600.00 a month. I trusted them to have the answers within themselves. I apologized for having demeaned them by talking about what they could do, rather than learning how they heck they did do it. It was transforming,

> to say the least, as to what happened in the room after that: they all got to work telling, not just me, but everyone there, how they did do it. We then compiled a list of helpful budget hints, and everyone was extremely happy about the learning process. ("Getting Out of the Way," an asynchronous on-line conference sponsored by Literacy BC, May 23, 2001).

Meaningful, realistic problems do not always lend themselves to mathematical solutions; other factors may be more important. If a student lives with a drunk who spends all the money on booze, budgeting is not the solution to her life problem, and not even the solution to her money problem. Similarly, time management may not be a matter of estimating how long things take, filling in a schedule, learning to plan for future events like tests, and so on. If a student is on-call at two part-time jobs and she must say yes to any call, or if she has on-call responsibilities in her extended family, or if she has to stand in line for hours at the welfare office or the food bank, she does not control her own time. Because she does not control her own time, she cannot plan to get her homework done by filling in a schedule. Rather, the solution to her problem lies in gaining some control over her time.

I'm not suggesting that the math class is the place to take on how to deal with a drunken mate (although it may be), or how to get better working conditions or get other family members to help with family responsibilities (although these, too, may be appropriate). However, I am recognizing that math content that purports to offer real life situations often misses the mark, no matter how well meaning it is. So-called "real life" problems may not be any more relevant to students' lives than this one, from *Even Cowgirls Get the Blues*: "If a hen and a half lays an egg and a half in a day and a half, how long will it take a monkey with a wooden leg to kick the seeds out of a dill pickle?" (Robbins, 1981, p.16).

Real Problems Are Messy

Problems from real life do not fit neatly into 50-minute classes, nor do they build on math skills and concepts systematically. Because they require divergent thinking, and many different kinds of math skills, they do not fit neatly into the kinds of texts we usually use. Different students bring different math experience and different life experience to realistic problems, and this wide range of knowledge and ability adds to the messiness when we use them in the classroom. By using the ordinary kind of textbook problems, we control this messiness to some extent. "I've taught everyone how to find the percent one number is of another. So I'll give them this page of questions that ask them to find what percent one number is of another, and everyone will be able to do it." In order to use real problems in the classroom, we have to give up some of that control, and agree to the messiness of real life. Of course, that means that we will also have to deal with our own resistance to giving up a "neat" math class, and that we will have to deal with student resistance, and that somehow we will have to convince colleagues and administrators that we are doing real math.

Just Do It

Welcome students' problems. Ask them to bring them in, and get the whole class to work on them. Follow where they lead. Here are some examples of real problems that you might take up in a math class.

Doing Taxes

Sometimes everyone will have the same problem: income tax, for example. Every year, the government offers training for people who will volunteer to fill out simple tax forms for others. Take the training yourself, because there are parts of the tax system that don't apply to your life, or invite one of the trained volunteers in, set a time to give over the class to doing tax, and ask everyone to bring in their various T-4's and other pieces of paper. Walk through the process with everyone filling in his own numbers. Set up a time for private consultations with the tax volunteer too.

That being said, many students have their taxes done at a commercial tax firm because they offer an immediate refund, after charging a commission. Students may have to/want to pay the commission in order to get immediate cash; however, if they figure out their taxes in math class, and figure out the amount they will pay in commission and fees, they will make an informed decision about whether to mail it in themselves, and wait for the refund, or go to a commercial tax firm.

Field Trips

Below are some examples, which you can tailor to concepts you want to teach.

Take a walk around your area
Go with the students for a walk in the neighborhood, to bring math to real life.

- Find a building made of bricks or concrete blocks and ask them 1) to make an estimate of how many bricks or blocks are in one wall, and in the whole building; and 2) to find a defined area on the building where there are about 100 blocks or bricks, and another defined area where there are about 1 000 blocks or bricks. By defined area, I mean one that is easy to see, for example, "the whole side wall" or "the part of the wall to the left of the doorway" or "the blocks below the white line." Make a sketch of these areas, and label them. This is a way of establishing a mental picture of the size of 100 and 1 000.
- Ask students to make a list of the math problems that someone had to solve to make the street look the way it does. (How many tulip bulbs do I need to fit into the flowerbed at 30 cm apart? How many steps do I need to cut so the staircase will reach the door? How much lumber do I need to buy for this fence? How many panels for that fence? How much paint do I need to cover the walls of this house? What angle do I cut for the pitch of the roof?) Just list the questions—no answers required.
- Make a list of questions that students can answer about a given area, for example, a block, or a three-block strip of a street. Ask about such things as the number of single or multiple family dwellings or the kind of businesses

on the street; when the students have answered the questions by walking the area, ask them to categorize the data and figure out proportions or percentages. For example, in a business district, to sort all the businesses into categories (food, retail, financial, medical, legal, government, etc.) and then to write proportions or percents to describe their data. Finally, they can make charts or graphs to display the data. If you ask different students to go to different areas, they can compare their results.

Take a trip to the do-it-yourself store

Take students to the lumberyard or the home renovation store with a sheet of those questions you find in every text that teaches area and perimeter. Have them answer the questions using material found in the store. Take, for example, the question about how much flooring to buy for a particular size of room. They should pick out the flooring they want, figure out the area of the room in the problem, check that the units in their problem are the same as the units the flooring is sold by (square feet, square yards, square meters), and figure the cost of the flooring. Let them do the same thing for the problem about how many panels of fencing to buy. A digital camera is a welcome addition here, so that they can attach a picture of the flooring or the fence to their solution to the problem.

Call ahead to the store to let them know you're coming, and you may get co-operation and help from the staff at hand.

Find out about numbers at your program

Make a list of How much? How many? questions about your school, and split the class up into teams to go to various people to find the answers. Make notes of the numbers they find, and present them in graphic form to the class. The students will come up with some questions they are interested in, and then you can decide where to go to find the answers. Below are some examples, only.

- Some students could go on a tour of the facilities with the person in charge of facilities, for example, to find out about numbers in that job, such as, How much toilet paper does the school use in a year? How many photocopies get made in a month? How many cleaners work for how many hours a week/year?
- Other students could talk to an administrator to find out about how many instructors work there, how many support staff, how many administrators? Who has worked there the longest? Who has worked there the shortest time? How many students? What is the total budget the person administers?
- Other students might go to the cafeteria to find out how many cups of coffee are served per day, or how many hamburgers; what days are busiest, slowest; how many times the dishwasher is run, how much detergent is used, etc.

Go to the supermarket

Ask students, How do you shop? How do you decide what to buy or where or when to buy it? Do the stores charge more on welfare day? What are some questions you might like answered?

Some students may be banned from some stores, so ask the class what would be a good place to visit before you set out. Phoning ahead may get you some helpful cooperation from store staff.

Bringing Real Life to the Classroom

Sometimes issues of privacy and messiness make it difficult to ask students to bring their real life problems to math class. Well, life inside the classroom is also real, and everyone can see what the parameters are. Look around for problems at school that students could take part in solving. Where is your institution spending money that students might have some input? Some examples follow, but really, it is a question of keeping your ears and eyes open.

Decorating or renovating

It would be a total fantasy to ask everyone to imagine that they were going to spend $1000 decorating a room in their house, or even $200. Yet they can see that some part of the building where classes are held needs work—perhaps the foyer is shabby, or the coffee room doesn't have enough seating, or the smoke pit is damp and cold, or somewhere there is not enough storage, so boxes of various supplies are sitting around the hallways or classrooms.

Take a space in your program—the lobby or lounge area, the cafeteria or a classroom, or an outside area where students hang out, and ask them to make a plan for refurbishing it.

Ask them to make a scale diagram of the space and show placement of various elements they would bring in. Go on a field trip to price paint, hardware, furnishings, etc., with a camera so students can take pictures of things they would like to incorporate into their design. Back in class, ask them to figure out how much things would cost and present a budget and their design to their classmates. The class could decide to present some or all of their ideas to administrators, or to other student groups, although the administration is likely to be a more sympathetic audience, whether or not they actually agree to some of the changes.

Rice Krispies® 101[3]

If your class wants to raise money for any purpose, the process involves math from start to finish. Making and selling something is a time-honoured method of raising money. I'll use Rice Krispies® squares (recipe follows) as an example here, but many other recipes or objects would work in a similar way.

Here are some of the math connections that might come up as the class works on this moneymaking project:

- Estimate how many pieces you will be able to sell.
- Using the large batch recipe below, figure out how many batches will give the number of squares you estimate you will be able to sell.

[3] This idea came from a workshop/consultation with instructors at the Fundamental Articulation Committee Meeting, Abbotsford, 2006.

- Make a list of supplies and shop for them. Make a note of the cost of the supplies.
- Make the squares.
- Wrap each piece and make a label that gives nutritional information for each piece. (The large batch recipe uses whole packages of cereal and marshmallow, so it is a simple matter to add the information on the package labels. The information on the label of the butter or margarine will have to be calculated for the amount you use. Then divide by the number of pieces you cut.)
- Figure out the cost per serving, and decide how much you will charge.
- After the sale, compare your actual sales with estimated sales. Figure out your profit.
- How can you make them healthier? Try some small batches with a different kind of cereal (less sugar) or make your own marshmallows (less sugar) or ???

Kellogg's® Rice Krispies Treats® (Large Quantity Recipe)

Prep Time: 30 minutes **Servings:** 48

INGREDIENTS

1 1/2 Sticks (6 oz.) margarine or butter

3 pounds regular marshmallows

1 3/4 gallons ***Kellogg's® Rice Krispies®*** cereal (1 lb. 8 oz.)

DIRECTIONS

- In large saucepan or steam-jacketed kettle, melt margarine. Add marshmallows, stirring constantly, cook over low heat until marshmallows are melted. Remove from heat or turn off heat..
- Add KELLOGG'S® RICE KRISPIES® cereal to hot marshmallow mixture, stirring until cereal is well coated.
- Using buttered spatula or waxed paper, press mixture evenly in 26 x 18 x 1-inch sheet pan coated with cooking spray. Cut into 2 x 2-inch squares when cool. Best if served the same day.

Note For best results, use fresh marshmallows.

Diet, reduced calorie or tub margarine is not recommended.

Store no more than two days in airtight container.

7

So in a nutshell, for me the issue of "Getting out of the Way," and relinquishing power, starts with trust: trust in the learning process; trust that I can take control and let it go; trust that I know what I am doing and that the students, when given true power over their learning, will, for the most part, rise to the challenge and engage in the process. — Diana Twiss, WISH/Capilano College, April 2001, in "Getting out of the way," an asynchronous on-line conference sponsored by Literacy BC.

Students Take Charge

Often we talk about "empowering" learners, but that word doesn't ring true for me. It implies that there is something that I can do to learners that will make them powerful people. Instead, I have come to think about my role as getting out of the way, so that students can claim power over their learning. Instead of focusing on students and what they need to be "empowered," I have come to concentrate on myself. What barriers do I put in the way of students having power over themselves and their learning? What can I do to chip away at those barriers?

In 2001 I moderated an on-line discussion sponsored by Literacy BC called "Getting Out of the Way." The theme of the conference was power—the power we have as instructors, the power students have, and the dance of taking power, giving power and sharing power as individuals and in groups.

For six weeks we talked about our practice as instructors, tutors and administrators; we talked personally about "striking a balance," and about "walking a fine line" in our attempts to provide leadership while leaving room for students also to lead. We talked about the leadership that the teacher provides as part of the contract between teacher and student, and the leadership students take as they strive to make schooling work for them. That leadership, contrasted with control, was everywhere in the discussion, as people strove for balance. As instructors, we have expertise and experience that makes us valuable leaders, but when that expertise and experience give us control over the teaching-learning process, and leave the students with little or no control, there we are, back in that old situation, with us at the front telling students what to do, and students lying back in their seats, arms folded, saying, “So, teach me. (If you can!)”

Strategies

Make Your Practice Transparent

Information is power. I try to share my knowledge and experience and my expertise as a teacher so students can act on it, not because I say so, but because they have chosen on the evidence to do so. My job is not to “do them good,” but rather to share what I know about how to learn math so they can make their decisions about their own learning.

Help Them Learn about Themselves

There are many on-line and pencil-and-paper inventories that will help students understand their own learning styles and their areas of strength. If you have access to counseling services, there may be someone there who can work with individuals or with the class as a whole to help people figure themselves out. I like to do more than one inventory, so that students can compare the results. Some of the inventories listed below give results in the shape of a graph, which leads to interesting comparisons from one student to another. After the students have a good idea of their strengths, go on to work with them to design assignments or study plans that work best for them.

Multiple Intelligences test: gives a graphic print out; uses plain language terms as well as regular, for example Linguistic Intelligence is also called Word Smart.
http://www2.bgfl.org/bgfl2/custom/resources_ftp/client_ftp/ks3/ict/multiple_int/index.htm

Multiple Intelligences inventory: a checklist, shorter than the inventory above.
http://snow.utoronto.ca/prof_dev/tht/multint/content/miref.html

Learning styles: gives a graphic printout http://www.learning-styles-online.com/inventory/default.asp?ref=ga&data=learning+styles+free+test

Learning styles: a checklist shorter than the one above
http://www.metamath.com/lsweb/dvclearn.htm

Give up the Power to Make Decisions

To solve a math problem, or to do some computation, you have to make decisions: Of all this information, what is relevant? What features of this problem are similar to ones I’ve solved before? What do I really want to find out? How shall I begin? Do I put the big number on top or on the bottom? One of my goals as an instructor is to put learners in charge of making decisions. How do I shape my practice to accomplish this goal? Following are some answers to this question.

I give learners opportunities to make decisions.
The simplest example is to ask students to plan—to make a plan for the part of the class session where they do independent work, for their study outside of class, for how they will get help when they need it—anything connected to their study of math. When I ask, “What’s your plan for …?” the learner knows from experience and from previous talk

that I expect his activities will not be random, and that I expect him to be making a plan in the light of what work he has to do, his personal style, his energy level and what constraints he has on his time.

I am clear with myself about my expectations.
If I am to be successful in turning decision-making power over to learners, I need to be clear with myself about my own expectations around learner decisions. I expect them to make a decision about how they will spend their time, for example, but I do not expect them necessarily to make a "good" decision. Furthermore, I expect them to notice that they have made a decision, rather than me making it for them.

Setting aside my judgments about the efficacy or correctness of learner decisions is the hardest part of all. Of course, when learners make "good" decisions ("I'm going to finish this assignment before I leave class, and review the chapter at home tonight,") I don't have any trouble "letting" them make the decisions. When they make "bad" decisions I have to take myself in hand. I have to remember that I expect them to make decisions, and that when a learner makes a decision, she has fulfilled the expectation. When Rhonda says to me, "I'm going to skip my math class today so I can make a birthday card for my sister," I say, "I see you've made a plan. Do you know where the fancy paper is?"

At the end of the week, when she has not completed her work in math, or when she is talking about how to move to the next level of math, I will remind her of the criteria for completing my class. "You didn't get your assignment done this week. What could you do to make sure you get it done next week?"

I emphasize the act of decision-making
But in the moment, Rhonda's job is to make a decision, and she has decided to make the birthday card instead of going to math class. If I say, "Can't you make the card later? Do you want to miss class (again)?" then I am making the decision, not Rhonda, and my goal to have learners make decisions is thwarted. If I fight Rhonda's decision, she may continue to refuse to go to class and make a scene, which wastes a lot of my time and sets a sour atmosphere in the room, which I have to work to make right. Alternatively, she may come to class reluctantly and not participate fully because of the emotions left over from my refusing her decision to skip class, which wastes her time and sets a sour atmosphere between us, which I have to work to make right.

Later in the day, when she has to leave school early, she may blame me as she goes home without the birthday card for her sister. She made her decision to skip class knowing she had to leave school early, but, although I had less knowledge (I didn't know she had to leave early), I had more power. It was easier for her to agree with my decision and then blame me later, when it turned out to have been the wrong one for her.

When there are decisions to be made about what learners will or won't do, I think the most fruitful thing I can do is accept the learner's decision, and bring the fact of making the decision to the foreground. "I see you've made a plan."

I notice out oud when learners make decisions.
Most decisions made in math class are not life changing. Should I use a pencil or a pen? Should I solve this proportion in four steps or three? Should I cancel here or multiply across and reduce the answer? Should I come late to class after my doctor's

appointment, or just go home? How can I get help with this computer program? The consequences of making the wrong decision are not great, and the increase in self-confidence and sense of control over the learning process far outweigh any consequences of making the wrong one.

On the other hand, I have an opinion about all of these things. If called on to make a decision about them, I can quickly decide which would be the better course in a given situation. My job is to refuse to make those decisions; my job is to get out of the way so the learner can make them, and reinforce the learner's confidence in his ability to make decisions that work for him. When a learner comes and says, "What shall I do?" I say, "Your choice. What are you thinking about?" He can outline his thinking with me as an audience, perhaps asking a question for clarification, but at the end of it, I say, "Your decision. What will you do?"

I model my own decision making.
One way to bring decision making out of the "hidden" curriculum and into the forefront is to make my own process transparent. Many parents have told me that they had very little idea about what being a parent entailed until they had their first child; in the same way, most students have very little idea about what decisions a teacher makes; indeed, I had no idea until I became a teacher myself. Because I know that so many of my decisions are hidden from my students, I talk about how and why I decide to present things in certain ways, explain the department thinking on test policies, and so on.

I also tell learners how their plans influence my planning. When Rita says, "I'm going to get a coffee, then could you look at my work?" I can say, "Okay. I should be finished working with George by then. If you're back in 10 minutes, I'll mark it for you. If not, it'll have to wait, because I have to get on to my next class."

I give a lesson in saying "No!"
Of course, when everyone agrees about what should be done, there is no problem making a decision. It is when there is no agreement that we want learners to make decisions for themselves. Asserting that decision may be difficult, especially when a learner has to assert it in an area usually controlled by the teacher.

I have developed a workshop session on saying no; it goes on the schedule every term as "How to say 'No,' to the teacher," and learners are free to attend or not. In the workshop, learners get a chance to express some of their feelings about saying no to someone in authority, and are given explicit instructions and role play in saying no in situations that come up in class. Their assignment is to say no to a teacher at least once in the following week, and much public acclaim comes as learners say no to a request I make. This explicit lesson makes it clear in the public space of the classroom that it is okay to decide not to go along with a suggestion from the teacher; that making your own decisions is expected; and that saying no will be respected. It also gives me time to say publicly that I can deal with people saying no to me. If I need something done, I may ask a learner to do it; if the answer is no, I'll ask someone else, or make some other plan to get it done, but I don't take it personally if someone says no to me, and I won't lay on a guilt trip when they do. Again, this making public my own decision-making process provides an explicit model for learners.

8

Activities for Students

The activities in this section are not meant to be a complete teaching method for these topics, rather they are examples of the kinds of activities instructors can make up to go with whatever text they are using, or whatever other lessons they are teaching. They are arranged here according to the subject matter of the activities. In each content area, there are group activities and activities using manipulatives or visual representations of some kind. Each section has an introduction with suggestions for using the activities in the section.

Place Value

There are two kinds of activities here: in the first, students use manipulatives to model or demonstrate for themselves some of the ideas inherent in the place value system, and in the second, students work in groups at the board. The ideas in one set practice and reinforce the ideas in the other set; there is a lot of practice and review involved in each, and a measured but quite quick movement towards more complex mathematical ideas.

I have used both these sets of activities successfully for many years, but would refer the reader to the discussions of student resistance and of group work in Chapters 2 and 5 to ensure a secure underpinning for these activities.

Place Value Demonstrations

The activities on the following pages require students to use math tools (manipulatives) to demonstrate their understanding of the place value system. Either commercial or home-made tools may be used. Each page might take about half an hour, and the same kinds of questions appear on every page, although they are more complex in later pages than in the earlier ones. Students may use the math tools for help with any of the questions, but the checkmark symbol ✔____ next to a question means that the answer must be demonstrated with the tools. As students work on the activities, the instructor can circulate and sign off demonstrations as students get them ready, and mark the answers to questions not requiring demonstrations. Encourage students to compare answers with one another, and, in the case of a disagreement, to use the tools to figure out who is right.

Signing off the Demonstrations

✔____ Wherever this symbol appears on the page, students should use the tools to demonstrate that their answer is correct. Your job is to look at their demonstrations, ask questions to clarify or extend their understanding, and sign or initial on the line when you are satisfied. Encourage students to line up several demonstrations as they wait for you to circulate to their desks, rather than setting up one and waiting. Students can work individually or in pairs.

What should you look for?

Each type of question is shown below, with some suggestions for dealing with the demonstrations students offer.

Use the tools to show these numbers:

Check that the student has represented the number correctly. For example, if the number to be shown is 105, ask questions such as: "How did you decide what to use to show the 1? The 5?" "How did you handle the 0?" In the beginning, students will be less able to articulate their thinking, but as they work, their ability to talk "math talk" will increase. Any kind of talking is acceptable and valuable. Anything they notice, any pattern they see, shows that they are noticing patterns and beginning to articulate them.

Ask someone to set up some numbers for you with the tools:

The student must ask another student to set up the tools to show various numbers. You might give guidance about the difficulty of the questions, or just leave it to the students. You can sign off using questions similar to those above, or you can ask the student who set up the numbers to sign them off.

Put the decimal in these numbers:

Explain what the decimal does: "The decimal separates the whole number from the part number." Why bother with the decimal at this stage? Because we want students to name what place various digits occupy in a number. Ones, tens, hundreds, thousands, etc. are counted to the left of the decimal point, even if the decimal point is not written. Students will learn from this exercise that they can always put a decimal point in a number if they want it for any reason, and that the decimal point is always written to the right of the ones place.

Underline the digit in the ones place:

These and the questions which follow (tens place, hundreds place, etc.) help students make the transition from using tools to show the size of various digits to becoming aware that in writing numbers it is the position of the digit that shows its size. Sometimes the answer to the question "What are we counting?" helps here. The digit in the tens place counts 10's; the digit in the hundreds place counts 100's.

Use the tools to show each of these operations:

This important question allows students to demonstrate the meaning of add, subtract, multiply and divide. In each case, the operation is shown by the student's arm movements. "Add" is a pushing together of two amounts; "subtract" is a removing of one amount from another. Most students can show these easily; however, they have a harder time with multiply and divide.

"Multiply" can be shown by repeatedly adding an amount. 3 x 5, for example, can be shown in two ways, either by putting three blocks (or toothpicks) on the table five times, or by putting five blocks on the table three times. Often students will mistakenly get out three blocks and five blocks and add them together. The operation of multiply is in the arm movement—putting an amount on the table the required number of times.

Division is the trickier because of the arbitrary nature of writing the question; when we write 6 ÷ 2 we mean that six is to be divided up, not two. The operation of division is also in the arm movement; the second number controls the arm movement. 6 ÷ 2 can be shown in two ways:

two groups of three:

☐ ☐ ☐ ☐ ☐ ☐

Here the question is, if I have six things and want to share them between two people, how many will each person get?

OR
three groups of two:

☐ ☐ ☐☐ ☐ ☐

Here the question is, if I have six things, and give them out two at a time, how many people will get some?

Usually students will make the first demonstration; after they are easily able to talk about the operation of division in this form, I show them the other way, too. They will notice that the answer is the same, no matter which way you think of it.

Start with the first number, double it, double again, double at least five times:

The purpose of this question is to encourage students to do some mental math, using tools as necessary to help them. The ability to double numbers easily is useful for teaching addition facts and the times tables. Nearly everyone can double numbers up to 10, and many people can double most of the numbers up to 15, and many common numbers upwards of there. I think it is worth spending a little time to teach them to double mentally starting from the left. So to double 125, for example, I think "Double 100 is 200, and double 25 is 50, which makes 250." Double 156 is double 100 (200) plus double 50 (300) plus double 6 (312).

Place Value Demonstrations

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Use the tools to show these numbers:

9 19 29 59 109 119 ✔ _____

Ask someone to set up five numbers for you with tools. Write the numbers here:

_______ _______ _______ _______ _______ ✔ _____

Put the decimal in these numbers:

20 17 260 2 591 47

Underline the digit in the ones place:

11 256 779 502 89

Underline the digit in the tens place:

21 46 79 123 459 ✔ _____

Underline the digit in the hundreds place:

135 168 789 1 981 2 098 ✔ _____

Use the tools to show each of these operations. Highlight the two that are the same:

3 – 2 3 x 2 3 + 3 3 ÷ 3 ✔ _____

Start with the first number, double it, double again; double at least five times:

2 _______ _______ _______ _______ _______

3 _______ _______ _______ _______ _______

5 _______ _______ _______ _______ _______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Use the tools to show these numbers:

7 22 66 666 1 000 ✔ ______

Ask someone to set up five numbers for you with tools. Write the numbers here:

_______ _______ _______ _______ _______ ✔ ______

Put the decimal in these numbers:

35 15 22 159

Underline the digit in the ones place:

79 44 289 7 945 690

Underline the digit in the tens place:

17 36 459 793 259 1 267 ✔ ______

Underline the digit in the hundreds place:

890 1 089 356 1 256 679 347

Underline the digit in the thousands place:

4 135 16 088 3 789 25 981 120 098

Show each of these operations. Highlight the two that are the same:

3 + 5 5 + 5 + 5 3 x 5 5 ÷ 5 ✔ ______

Start with the first number, double it, double again, double at least five times:

20 _______ _______ _______ _______ _______

3 _______ _______ _______ _______ _______

30 _______ _______ _______ _______ _______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Use the tools to show these numbers:

37 33 789 350 20 1 111 ✔ ______

Ask someone to set up some numbers for you with tools. Write the numbers here:

_______ _______ _______ _______ ✔ ______

Put the decimal in these numbers:

321 30 440 1 700

Underline the digit in the tens place:

32 378 1 005 $27.63 $126.98

Underline the digit in the hundreds place:

670 1 068 468 2 349 789 376

Underline the digit in the thousands place:

8 900 321 089 3 567 1 256 679

Use the tools to show each of these operations. Highlight the two that are the same:

4 x 4 4 ÷ 4 4 – 4 4 + 4 + 4 + 4 ✔ ______

Start with the first number, double it, double again, double at least five times:

3 _______ _______ _______ _______ _______

4 _______ _______ _______ _______ _______

10 _______ _______ _______ _______ _______

20 _______ _______ _______ _______ _______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Use the tools to show these numbers:

10 11 100 101 110 111 1 111 ✔ _____

Ask someone to set up some numbers for you with tools. Write the numbers here:

______ ______ ______ ______ ✔ _____

Put the decimal in these numbers:

1 304 798 1 000

Underline the digit in the ones place:

$48.21 $627.89 26.1 27.89

Underline the digit in the tens place:

10 305 $276.10 $73.95 73.967 173.01

Underline the digit in the hundreds place:

1 789 $100.78 $321.98 34 905 204.1

Underline the digit in the thousands place:

67 000 10 068 4 680 25 349 789 376

Show each of these operations. Highlight the two that are the same:

200 + 2 200 ÷ 2 200 x 2 200 + 200 ✔ _____

Start with the first number, double it, double again, double at least seven times:

3 ______ ______ ______ ______ ______ ______ ______

5 ______ ______ ______ ______ ______ ______ ______

7 ______ ______ ______ ______ ______ ______ ______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Show these numbers:

1004 4010 10 000 11 101 ✔ ______

Ask someone to set up some numbers for you with tools. Write the numbers here:

_______ _______ _______ _______ ✔ ______

Put the decimal in these numbers:

44 2 1 798 45

Underline the digit in the tens place:

235 10 695 $179.23 158.9325 21.6789

Underline the digit in the hundreds place:

$2 356.97 327 327.1 5 078.92

Underline the digit in the thousands place:

$1 000.78 $3 291.98 6 934 905 2 704.1 21 567

Underline the digit in the ten thousands place:

34 135 616 088 23 789 7 825 981 102 098

Show each of these operations. Highlight the two that are the same:

10 x 2 10 + 10 10 – 2 10 ÷ 2 10 + 2 ✔ ______

Start with the first number, double it, double again, double at least seven times:

2 _______ _______ _______ _______ _______ _______ _______

5 _______ _______ _______ _______ _______ _______ _______

7 _______ _______ _______ _______ _______ _______ _______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Underline the digit in the thousands place:

$2 356.97   32 789   43 275.5   456 078.92

Underline the digit in the ten thousands place:

38 900   321 089   63 567   1 789 356

Underline the digit in the hundred thousands place:

176 365   459 793   259 259 259   1 026 746

Underline the digit in the millions place:

1 625 216   4 655 255   21 798 967   6 123 459

Show each of these operations. Highlight the two that are the same:

1 000 + 1 000 + 1 000   1 000 – 3   1 000 x 3   1 000 ÷ 1   ✔ ______

Start with the first number, double it, double again, double at least seven times:

30 _______ _______ _______ _______ _______ _______ _______

7 _______ _______ _______ _______ _______ _______ _______

11 _______ _______ _______ _______ _______ _______ _______

Add 10 to each of these numbers:

23   47   29   83   77   14

✔ ______

Subtract 10 from each of these numbers:

93   47   29   13   77   14

✔ ______

A check mark tells you to use the tools to show your answer. Your teacher will look at your work with the tools and sign beside the check mark. You can use the tools to help you with any work on this page.

Underline the digit in the ten thousands place:

25 000 107 068 40 680 12 327 349 789 376

Underline the digit in the hundred thousands place:

32 335 378 1 005 256.89 $277 256.63 $3 006 126.98

Underline the digit in the millions place:

8 176 365 459 607 793 259 259 259 10 026 746

Start with the first number, double it, double again, double at least seven times:

50 _______ _______ _______ _______ _______ _______ _______

7 _______ _______ _______ _______ _______ _______ _______

11 _______ _______ _______ _______ _______ _______ _______

Add 10 to each of these numbers:

123 547 2 229 383 27 19 ✔ _____

Subtract 10 from each of these numbers:

123 547 2 229 383 27 19 ✔ _____

Add 100 to each of these numbers:

245 367 28 4 567 1 789 798 ✔ _____

Subtract 100 from each of these numbers:

245 367 28 4 567 1 789 798 ✔ _____

Whole Numbers at the Board

Time: 15-20 minutes a day

The following series of whole number activities allows you to check how fluent your students are with reading and writing numbers to one million, and gives students lots of practice in reading numbers and in working with place value. It also shows the value of review and over-learning; students will notice as it gets easy to answer questions similar to the ones that were difficult the week before. Students should find most of these questions easy, because they are doing many examples of similar questions until it seems easy. If every question is hard, they will be reluctant to go to the board. Every day only a few questions should present some challenge. You can tailor the questions to suit your students, backing up a step when they are having difficulty, and asking them to explain their thinking on questions they are finding easy to do.

When students can give a correct response quickly and easily, that is the time to ask them to explain their thinking, since usually the skill of talking about math lags behind the skill of doing math. So while some students are finding a particular exercise challenging to do, others, who find it easy to do, can practice the thing that is challenging to them, that is, explaining their thinking.

When students are familiar with the tasks, ask for a volunteer to "be the teacher" and read the instructions for other students to follow. The challenge of checking that student responses are correct, while running the group process of reading answers and noticing patterns, will be an interesting math experience for some students whose skills in doing the math are more advanced than most of the group.

Process

Each day, ask students to go the board to read and write some numbers which you will call out. Encourage them to look around at other students' work, stand beside someone who is good at math, get a long piece of chalk, and so on. They should do whatever they need to be comfortable at the board. Especially, make sure there is an eraser between every pair of students. If a student has to ask for an eraser, he calls attention to his error. We want him to take a risk, so make sure he can quickly and quietly erase his answer if it is wrong.

Every day, the work follows the same pattern:

1. As you call out the first instruction, students will write the number at the board. For example, "Write a three-digit number with 5 in the ones place." There will be a variety of correct answers.
2. Quickly look around and make sure that every response is correct, helping individuals as necessary.
3. Ask one of the students to read the number written by the student on their right; make sure it is loud enough for everyone to hear.
4. Ask the next student to read the number written by the student on their right, and so on until every number has been read.

5. Notice, comment on or ask about any patterns you see. For example, which number is largest? Smallest? What is the largest possible correct answer? Smallest possible? When many students write the same number, is it because they don't have many choices of correct answers? (For example, write the largest possible three-digit number.)
6. Go on to the next instruction, and the next.
7. Finally, thank students for their participation and their excellent work, and invite them to sit down.

Day 1

- Rule out the use of 0 as a first digit in today's work.
- Write a one-digit number that is bigger than 5. Ask each student to read the number written by the student on the right. Ask: Who wrote the smallest number? Who wrote the largest number? And so on as suggested above.
- Write a one-digit number smaller than 7. Ask each student to read the number written by the student on the right. Questions as above.
- Write a two-digit number with 7 in the tens place. Ask each student to read the number written by the student on the right. Questions as above.
- Write a two-digit number that comes between 17 and 25. Ask each student to read the number written by the student on the right. Questions as above.
- Write a two-digit number with a 0 in it. Ask each student to read the number written by the student on the right.
- Write a three-digit number with 9 in the hundreds place. Ask each student to read the number written by the student on the right.
- Write a three-digit number that comes between 127 and 130. Ask each student to read the number written by the student on the right.
- Write a three-digit number with a 0 in it. Ask each student to read the number written by the student on the right.
- Write a three-digit number with two zeros in it. Ask each student to read the number written by the student on the right.
- Ask students to sit down.

Day 2 (Preparation: Review or teach reading numbers larger than 1 000.)

- Rule out the use of 0 as a first digit in today's work.
- Write a three-digit number with 5 in the tens place. Ask each student to read the number written by the student on the right. Ask questions of the whole group, for example: Which number is largest? Smallest? What is the largest possible correct answer? Smallest? When many students write the same number, is it because they don't have many choices of correct answers?

- Write a three-digit number with 5 in the hundreds place. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a three-digit number with 5 in the ones place. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number with 7 in the tens place. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number that comes between 1 700 and 1 800. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number with three zeros in it. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number with two zeroes in it. Ask each student to read the number written by the student on the right. Ask questions, as before.
- Write a four-digit number with a 0 in it. Ask each student to read the number written by the student on the right. Ask questions, as before.
- I will give you two digits, 8 and 3. Write all the numbers you can, using those two digits. Write the numbers in order from smallest to biggest.
- Thanks. You may sit down again.

Day 3

- As usual, rule out the use of 0 as a first digit in today's work.
- Write a four-digit number with 7 in the thousands place. Ask each student to read the number written by the student on the right. Ask questions of the whole group, for example: Which number is largest? Smallest? What is the largest possible correct answer? Smallest? When many students write the same number, is it because they don't have many choices of correct answers?
- Write a five-digit number. Ask each student to read the number written by the student on the right. Ask questions, as usual.
- Write a five-digit number with 7 in the ones place. Ask each student to read the number written by the student on the right.
- Write a five-digit number that comes between 17 000 and 18 000. Ask each student to read the number written by the student on the right.
- Write a six-digit number with a 0 in it. Ask each student to read the number written by the student on the right.
- Write a six-digit number bigger than 500 000. Ask each student to read the number written by the student on the right.

- Write a six-digit number with 4 in the thousands place. Ask each student to read the number written by the student on the right.
- I will give you two digits, 6 and 4. Write all the numbers you can, using those two digits. Write the numbers in order from smallest to biggest.
- I will give you three digits, 5, 6 and 9. Write all the numbers you can, using those three digits. Write the numbers in order from smallest to biggest.
- Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 2.
- Thanks. You may sit down again.

Day 4

- Again, and on all the following days, rule out the use of 0 as a first digit in today's work.
- Your questions for the first set could focus on explaining the thinking that led to the answer. How did you decide on your answer? What would the next number bigger (or smaller) be? Does that meet the criteria? Looking at all the answers, what pattern in 0 and 9 do you see?
- Write the biggest one-digit number possible. Ask someone to read it.
- Write the smallest two-digit number possible. Ask someone to read it.
- Write the biggest two-digit number possible. Ask someone to read it.
- Write the smallest three-digit number possible. Ask someone to read it.
- Write the biggest three-digit number possible. Ask someone to read it.
- Write the smallest four-digit number possible. Ask someone to read it.
- Write the biggest four-digit number possible. Ask someone to read it.
- Write the smallest five-digit number possible. Ask someone to read it.
- Write the biggest five-digit number possible. Ask someone to read it.
- Write a six-digit number with 7 in the thousands place. Ask each student to read the number written by the student on the right.
- Write a three-digit number bigger than 995. Ask each student to read the number written by the student on the right.
- Write a seven-digit number with 6 zeros in it. Ask each student to read the number written by the student on the right.
- Write a seven-digit number. Ask each student to read the number written by the student on the right.
- Write a seven-digit number larger than the one you just wrote, if possible. Ask each student to read the number written by the student on the right.

- I will say a number. Write the next number bigger than the one I say. 9 19 29 39 59 79 99 199 299 399 499 599 699 799 899 999
- I will give you two digits, 1 and 9. Write all the numbers you can, in order from smallest to largest.
- I will give you three digits, 7, 2 and 5. Write all the numbers you can, using those three digits. Write the numbers in order from smallest to biggest.
- Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 2.
- Repeat, starting with 3.
- Thanks. You may sit down again.

Day 5

- Write the smallest two-digit number possible. Read it to the person next to you.
- Write the smallest three-digit number possible. Read it to the person next to you.
- Write the smallest five-digit number possible. Read it to the person next to you.
- Write the smallest seven-digit number possible. Read it to the person next to you.
- Write the smallest eight-digit number possible. Read it to the person next to you.
- Write a three-digit number with 3 in the tens place. Read around.
- Write a four-digit number bigger than 4 000. Read around.
- Write a six-digit number. Read around.
- Write a seven-digit number. Read around.
- I will say a number. Write the next number smaller than the one I say. 8 29 31 40 50 80 100 200 400 500
- I will give you three digits, 1, 2 and 3. Write all the numbers you can, using those two digits. Write the numbers in order from smallest to biggest.
- I will give you four digits, 3, 7, 1 and 5. Write all the numbers you can, using those digits. Write the numbers in order from smallest to biggest. (This is much harder than the previous question; you might want to give some students four digits and others only three.)

- Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 2.
- Repeat, starting with 3.
- Thanks. You may sit down again.

Day 6

- Write the smallest seven-digit number possible. Read it to the person next to you.
- Write a three-digit number with 9 in the hundreds place. Read around.
- Write a four-digit number between 8 000 and 9 000. Read around.
- Write a three-digit number between 125 and 128. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 2.
- Repeat, starting with 3.
- Repeat, starting with 11.
- Thanks. You may sit down again.

Day 7

- Write the smallest seven-digit number possible. Ask someone to read it.
- Write a three-digit number with 9 in the hundreds place. Read around.
- Write a five-digit number between 29 000 and 30 000. Read around.
- Write a six-digit number between 200 000 and 300 000. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 20.
- Repeat, starting with 30.
- Repeat, starting with 11.
- Thanks. You may sit down again.

Day 8

- Write the largest seven-digit number possible. Read around.

- Write a five-digit number with 9 in the hundreds place. Read around.
- Write a five-digit number between 54 000 and 55 000. Read around.
- Write any number with 1 in the thousands place. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 7. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 2.
- Repeat, starting with 3.
- Repeat, starting with 11.
- Thanks. You may sit down again.

Day 9

- Write the largest three-digit number possible.
- Write any number with 9 in the thousands place. Read around.
- Write a number between 4 857 and 4 859. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 5. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 11.
- Repeat, starting with 13.
- Thanks. You may sit down again.

Day 10

- Write the largest six-digit number possible. Read it to the person next to you.
- I'll give a number; you write the next whole number bigger. 17 39 109 999 1 045 10 000
- Write a number between 587 and 600. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 2. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 5.
- Repeat, starting with 3.
- Thanks. You may sit down again.

Day 11

- Write the largest seven-digit number possible. Read it to the person next to you.
- I'll give a number; you write the next whole number bigger. 33 59 709 9 999 1 045 100 000
- Write a number between 1 110 and 1 115. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 3. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 7.
- Repeat, starting with 9.
- Thanks. You may sit down again.

Day 12

- Write the largest seven-digit number possible. Read it to the person next to you.
- I'll give a number; you write the next whole number bigger. 38 92 799 1 000 9 999 100 000
- Write a number between 1 256 and 1 260. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 2. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 11. Repeat, starting with 13.
- Thanks. You may sit down again.

Day 13

- Go to the board.
- Write the largest four-digit number possible. Read it to the person next to you.
- I'll give a number; you write the next whole number bigger. 56 99 709 9 999 200 000 1 000 000
- Write a number between 12 600 and 12 700. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 10. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 100.
- Repeat, starting with 9.

- I will give you a number. Cut it in half, then cut the answer in half and keep on going until you come to a number that you can't cut in half evenly. Start with 1 000.
- Repeat, starting with 1 200.
- Repeat, starting with 800.
- Thanks. You may sit down again.

Day 14

- Go to the board.
- Write the largest three-digit number possible. Read it to the person next to you.
- I'll give a number; you write the next whole number bigger. 909 127 6 425 9 999 7 777 99 000
- Write a number between 11 700 and 11 800. Read around.
- Write a hard number to read. Read around.
- Doubling: Start with 200. Double it, then double the answer, then double that answer. Keep going until the time is up. (Allow about two minutes.)
- Repeat, starting with 50.
- Repeat, starting with 30.
- Thanks. You may sit down again.

Day 15 (Preparation: Teach or review short method of multiplying by 10.)

- Go to the board.
- Multiply by 10. 75 155 250 2 500 17.9 26.7 27.89 156.9876
- Divide by short division. Start with 240; divide by 2 and keep dividing the answer until it can't go evenly any more.
- Repeat, starting with 1 000.
- Repeat, starting with 1 400.
- Thanks. You may sit down again.

Day 16

- Go to the board.
- Multiply by 10. 35 127 360 7,350 27.2 39.6 49.89 257.3694
- Divide by short division. Start with 650; divide by 2 and keep dividing the answer until it can't go evenly any more.

- Repeat, starting with 96.
- Repeat, starting with 84.
- Thanks. You may sit down again.

Day 17 (Preparation: Teach or review short method of dividing by 10.)

- Multiply by 10. 46 777 365 5 600 26.9 179.2 139.89 1256.3742
- Divide by 10. 30 120 50 100 900
- Thanks. You may sit down again

Day 18

- Multiply by 10. 29 760 127.3 130.35 127.96 3.7524
- Divide by 10. 10 25 100 120 30.1 25.20
- Thanks. You may sit down again.

Following Days

Many days of board work follow this pattern—multiplying and dividing by 100; short and long division, finding factors, and so on. Working at the board is a way to review, practice and consolidate work presented in tests and lessons.

Fractions

There are two kinds of activities here: in the first, students use manipulatives to demonstrate for themselves some concepts and operations with fractions, and in the second, students work in groups at the board. The ideas in one set practice and reinforce the ideas in the other set; there is a lot of practice and review involved in each, and a measured but quite quick movement towards more complex mathematical ideas.

I have used both these sets of activities successfully for many years, but would refer the reader to the discussions of student resistance and of group work in Chapters 2 and 5 to ensure a secure underpinning for these activities.

Fractions Demonstrations

The activities on the following pages require students to use math tools (manipulatives) to demonstrate their understanding of fractions and to show their answers are correct. Some students will find the answers to the questions first by using math notation and algorithms they know; when they go to demonstrate that their answer is correct, they may find that, indeed, it is not correct, and may then be in a position to rethink their algorithm, or to come to a clearer understanding of what the abstraction of the algorithm conceals. The manipulatives give them a chance to correct their mistake before it gets marked wrong by the instructor. Correcting their own errors and showing they are right increases confidence and understanding.

Other students, who do not know the algorithms, or who have less faith in their memory of them, will go directly to finding the answers simply by manipulatives. As they learn the algorithms from their texts or in class, they will begin to combine the two methods, with the learning from the experience with manipulatives helping them remember and understand the algorithms.

The checkmark symbol ✔____ next to a question reminds the student that the answer must be demonstrated with the tools. As students work on the activities, the instructor can circulate and sign off demonstrations as students get them ready. Encourage students to compare answers with one another, and, in the case of a disagreement, use the tools to figure out who is right.

Signing off the demonstrations

✔____ Wherever this symbol appears, students should prove their answer is correct by setting up a demonstration with the math tools. Your job is to look at their demonstrations, ask questions to clarify or extend their understanding, and sign or initial on the line when you are satisfied. Encourage students to line up several demonstrations as they wait for you to circulate to their desks, rather than setting up one and waiting. Students can work individually or in pairs.

What should you look for?

Each question is shown below, with some suggestions for dealing with the demonstrations students offer.

1. **Write these fractions in order, smallest to largest.**
   This question is an example of the usefulness of manipulatives—if the student writes the answer incorrectly before doing the proof, the manipulatives will show him what the correct answer is, and he will change his written answer to line up with the proof before you get there. You can come along and sign off on the demonstration without having to mark anything wrong, and can then engage in a conversation about how counter-intuitive the structure is, and have some conversation about the meaning, i.e., that the bottom number tells how many pieces the whole thing is divided into, and if you cut more pieces, each piece will have to be smaller.

2. **Circle the smaller fraction.**
   Ask, "How can you tell by the way the fractions are written which one is smaller?" Look for an answer like "A bigger bottom number means the whole is cut into more pieces, so each piece is smaller.

3. **Circle the smaller fraction.**
   Ask, "How can you tell by the way the fractions are written which one is smaller?" An answer like "When the bottom numbers are the same, you know the pieces are the same size, so you can just look at the top number to see how many pieces there are," means that the student is beginning to see the necessity of a common denominator when comparing fractions.

4. **Write a fraction that equals one whole.**
   Ask, "How can you tell by the way the fraction is written that it is equal to 1?" You are asking the student to articulate the fact that a fraction with the same top and bottom number always equals 1. (Except 0.) The manipulatives give the student a chance to find this out for himself, rather than hearing it from you.

5. **Write a fraction that equals one half.**
   Ask, "How do you know your answer is right?" and expect a variety of answers on the theme of "The top number is half of the bottom number."

6. **Write a fraction that is the same size as the one given.**
   Showing these equivalents for frequently used fractions helps to build up a visual memory that students can recall when they use standard methods from the texts to write equivalent fractions.

7-8. Put these fractions in the right space on the chart below.
   These questions help students get a sense of the size of different fractions. Ask, "How do you know this is in the right place?"

9. **Write these fractions in order from smallest to biggest. 7/8, 1/12, 5/4, 3/3.**
   By using the skills from the previous two examples, students can do this question without having to change everything to a common denominator. 1/12 is less than half, 7/8 is more than half, 3/3 is equal to 1 and 5/4 is larger than 1. Nice to show that an understanding of fractions can help students do mentally what would take a lot of work and the possibility of many errors to do by the standard method of changing to a common denominator.

11. Circle the question if the answer will be more than 1.

This skill can help with estimating answers to addition and subtraction questions before you do them, or with making a quick check to see if an answer is reasonable after doing an adding or subtracting question. Ask, "How do you know your answer is right?"

12-19.

Students may use these demonstrations as a way of understanding whatever algorithm they use to do questions of these types.

Fraction Demonstrations

A check mark tells you to use the tools to show your answer. Show your work with the tools to the teacher, who will sign beside the check mark.

1. Write these fractions in order, smallest to largest.

$\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ ✔ _____

When the bottom number is **smaller,** the fraction is ________________.

When the bottom number is **bigger**, the fraction is ______________.

2. Circle the smaller fraction.

$\frac{1}{2}$ $\frac{1}{12}$ ✔ _____ $\frac{1}{6}$ $\frac{1}{3}$ ✔ _____

$\frac{1}{4}$ $\frac{1}{12}$ ✔ _____ $\frac{1}{5}$ $\frac{1}{8}$ ✔ _____

When the bottom number is _____________**,** the fraction is **smaller**.

When the bottom number is ______________, the fraction is **bigger**.

3. Circle the smaller fraction.

$\frac{1}{12}$ $\frac{3}{12}$ ✔ _____ $\frac{2}{10}$ $\frac{1}{10}$ ✔ _____

$\frac{4}{12}$ $\frac{2}{12}$ ✔ _____ $\frac{1}{10}$ $\frac{3}{10}$ ✔ _____

When the bottom number is the **same**, the pieces are the ______________ size.

4. Write a fraction that equals one whole.

$1 = \frac{}{12}$ ✔ ______ $1 = \frac{}{2}$ ✔ ______ $1 = \frac{}{3}$ ✔ ______

$1 = \frac{}{5}$ ✔ ______ $1 = \frac{}{4}$ ✔ ______ $1 = \frac{}{6}$ ✔ ______

$1 = \frac{}{8}$ ✔ ______ $1 = \frac{}{10}$ ✔ ______ $1 = \frac{}{25}$

$1 = \frac{}{15}$ $1 = \frac{}{100}$ $1 = \frac{}{27}$

5. Write a fraction that equals one half.

$\frac{1}{2} = \frac{}{12}$ ✔ ______ $\frac{1}{2} = \frac{}{6}$ ✔ ______ $\frac{1}{2} = \frac{}{10}$ ✔ ______

$\frac{1}{2} = \frac{}{4}$ ✔ ______ $\frac{1}{2} = \frac{}{8}$ ✔ ______ $\frac{1}{2} = \frac{}{20}$

$\frac{1}{2} = \frac{}{24}$ $\frac{1}{2} = \frac{}{100}$ $\frac{1}{2} = \frac{}{1000}$

6. Write a fraction that is the same size as the one given.

$\frac{1}{2} = \frac{}{10}$ ✔ ______ $\frac{1}{4} = \frac{}{12}$ ✔ ______ $\frac{1}{3} = \frac{}{6}$ ✔ ______

$\frac{1}{2} = \frac{}{12}$ ✔ ______ $\frac{1}{4} = \frac{}{8}$ ✔ ______ $\frac{1}{3} = \frac{}{12}$ ✔ ______

$\frac{1}{2} = \frac{}{4}$ ✔ ______ $\frac{1}{4} = \frac{}{16}$ ✔ ______ $\frac{2}{3} = \frac{}{6}$ ✔ ______

$\frac{1}{2} = \frac{}{8}$ ✔ ______ $\frac{3}{4} = \frac{}{12}$ ✔ ______ $\frac{2}{3} = \frac{}{12}$ ✔ ______

$\frac{1}{2} = \frac{}{6}$ ✔ ______ $\frac{3}{4} = \frac{}{8}$ ✔ ______ $\frac{1}{6} = \frac{}{12}$ ✔ ______

$\frac{1}{2} = \frac{}{16}$ ✔ ______ $\frac{3}{4} = \frac{}{16}$ ✔ ______ $\frac{2}{5} = \frac{}{10}$ ✔ ______

$\frac{1}{5} = \frac{}{10}$ ✔ ______ $\frac{5}{6} = \frac{}{12}$ ✔ ______ $\frac{4}{5} = \frac{}{10}$ ✔ ______

7. Put these fractions in the right space on the chart below.

$\frac{4}{8}$ $\frac{11}{12}$ $\frac{3}{5}$ $\frac{1}{3}$ $\frac{5}{6}$ $\frac{1}{4}$ $\frac{4}{5}$ $\frac{5}{10}$ $\frac{9}{10}$ $\frac{1}{8}$ $\frac{2}{3}$

Less than 1/2	Equal to 1/2	More than 1/2	
			✔ ______

8. Put these fractions in the right space in the chart below.

$\frac{5}{8}$ $\frac{12}{12}$ $\frac{5}{5}$ $\frac{5}{3}$ $\frac{5}{6}$ $\frac{4}{4}$ $\frac{10}{10}$ $\frac{3}{5}$ $\frac{11}{10}$ $\frac{1}{8}$ $\frac{4}{3}$

Less than 1	Equal to 1	More than 1	
			✔ ______

9. Write these fractions in order from smallest to biggest.

$\frac{7}{8}$ $\frac{1}{12}$ $\frac{5}{4}$ $\frac{3}{3}$ ______________________ ✔ ____

$\frac{7}{6}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{3}{3}$ ______________________ ✔ ____

10. Reduce to lowest terms. This means to write an equal fraction with a bottom number as low as possible. For example: $\frac{2}{4} = \frac{1}{2}$

$\frac{2}{4} =$ ✔ ____ $\frac{2}{6} =$ ✔ ____

$\frac{6}{10} =$ ✔ ____ $\frac{3}{6} =$ ✔ ____

$\frac{4}{6} =$ ✔ ____ $\frac{8}{10} =$ ✔ ____

$\frac{4}{8} =$ ✔ ____ $\frac{2}{8} =$ ✔ ____

$\frac{3}{12} =$ ✔ ____ $\frac{5}{10} =$ ✔ ____

$\frac{6}{8} =$ ✔ ____ $\frac{2}{12} =$ ✔ ____

$\frac{6}{12} =$ ✔ ____ $\frac{4}{12} =$ ✔ ____

$\frac{8}{16} =$ ✔ ____ $\frac{4}{10} =$ ✔ ____

$\frac{8}{12} =$	✔ ______	$\frac{9}{12} =$	✔ ______
$\frac{4}{16} =$	✔ ______	$\frac{8}{16} =$	✔ ______
$\frac{12}{16} =$	✔ ______	$\frac{14}{16} =$	✔ ______
$\frac{5}{5} =$	✔ ______	$\frac{4}{4} =$	✔ ______
$\frac{2}{16} =$	✔ ______	$\frac{16}{16} =$	✔ ______

11. Circle the question if the answer will be more than 1. You do not have to give the answer.

$\frac{1}{2} + \frac{11}{12}$	✔ ______	$\frac{1}{2} + \frac{1}{6}$	✔ ______
$\frac{1}{2} + \frac{3}{10}$	✔ ______	$\frac{9}{10} + \frac{3}{4}$	✔ ______
$\frac{1}{2} + \frac{3}{8}$	✔ ______	$\frac{3}{4} + \frac{3}{4}$	✔ ______
$\frac{1}{2} + \frac{1}{24}$		$\frac{5}{8} + \frac{75}{100}$	
$\frac{1}{6} + \frac{1}{3}$	✔ ______	$\frac{1}{2} + \frac{7}{8}$	✔ ______

12. Add. Reduce the answer if you can.

$\frac{1}{2} + \frac{1}{2}$ ✔ ______ $\frac{1}{5} + \frac{3}{5}$ ✔ ______

$\frac{9}{10} + \frac{3}{10}$ ✔ ______ $\frac{1}{3} + \frac{2}{3}$ ✔ ______

$\frac{1}{2} + \frac{1}{4}$ ✔ ______ $\frac{5}{8} + \frac{1}{4}$ ✔ ______

13. Subtract. Reduce the answer if you can.

$\frac{3}{8} - \frac{1}{8}$ ✔ ______ $\frac{7}{12} - \frac{5}{12}$ ✔ ______

$\frac{9}{10} - \frac{3}{10}$ ✔ ______ $\frac{2}{3} - \frac{1}{3}$ ✔ ______

$\frac{1}{2} - \frac{1}{4}$ ✔ ______ $\frac{5}{8} - \frac{1}{4}$ ✔ ______

14. Add. Reduce the answer if you can.

$\frac{3}{8} + \frac{1}{4}$ ✔ ______ $\frac{7}{12} + \frac{1}{3}$ ✔ ______

$\frac{9}{10} + \frac{2}{5}$ ✔ ______ $\frac{1}{6} + \frac{1}{3}$ ✔ ______

$\frac{1}{3} + \frac{3}{4}$ ✔ ______ $\frac{1}{16} + \frac{3}{4}$ ✔ ______

15. Subtract. Reduce the answer if you can.

$\frac{7}{12} - \frac{1}{4}$ ✔ ____ $\frac{7}{12} - \frac{1}{3}$ ✔ ____

$1 - \frac{3}{4}$ ____ $\frac{2}{5} - \frac{1}{10}$ ✔ ____

$1 - \frac{1}{6}$ ✔ ____ $\frac{1}{2} - \frac{1}{4}$ ✔ ____

16. Add. Reduce the answer if you can.

$\frac{3}{4} + \frac{5}{8}$ ✔ ____ $\frac{7}{12} + \frac{5}{6}$ ✔ ____

$\frac{9}{10} + \frac{1}{10}$ ✔ ____ $\frac{2}{5} + \frac{7}{10}$ ✔ ____

$\frac{1}{2} + \frac{11}{12}$ ✔ ____ $\frac{5}{16} + \frac{7}{8}$ ✔ ____

17. Subtract. Reduce the answer if you can.

$1 - \frac{1}{8}$ ✔ ____ $\frac{1}{2} - \frac{1}{12}$ ✔ ____

$\frac{1}{2} - \frac{1}{4}$ ✔ ____ $\frac{2}{3} - \frac{1}{3}$ ✔ ____

$\frac{3}{4} - \frac{5}{8}$ ✔ ____ $\frac{4}{5} - \frac{7}{10}$ ✔ ____

18. Multiply. Remember, you can read "of" when you see "x."

$3 \times \frac{1}{8}$ ✔ ______ $\frac{1}{2} \times \frac{2}{3}$ ✔ ______

$10 \times \frac{1}{10}$ ✔ ______ $\frac{1}{4} \times \frac{8}{8}$ ✔ ______

$5 \times \frac{1}{2}$ ✔ ______ $\frac{1}{3} \times \frac{9}{12}$ ✔ ______

19. Divide. Divide the first number into pieces the size of the second number. How many pieces are there?

$1 \div \frac{1}{3}$ ✔ ______ $\frac{1}{2} \div \frac{1}{4}$ ✔ ______

$2 \div \frac{1}{5}$ ✔ ______ $\frac{6}{10} \div \frac{1}{5}$ ✔ ______

$\frac{2}{3} \div \frac{1}{3}$ ✔ ______ $\frac{1}{2} \div \frac{1}{8}$ ✔ ______

$\frac{5}{8} \div \frac{1}{8}$ ✔ ______ $\frac{1}{2} \div \frac{1}{10}$ ✔ ______

Fractions at the Board

Time: 15-20 minutes a day

The following series of activities with fractions gives students practice with fractions, allows you to check their understanding and their ability to manipulate fractions, and gives students a chance to articulate what they know. It also shows the value of review and over-learning; students will notice as it gets easy to answer questions similar to the ones that were difficult the week before. Students should find most of these questions easy, because they are doing many examples of similar questions until it seems easy. If every question is hard, they will be reluctant to go to the board. Every day only a few questions should present some challenge. You can tailor the questions to suit your students, asking more questions similar to ones they are learning, or skipping some questions that they are finding too easy or too difficult at the moment. For some students you can repeat one day's activities for several days, using different numbers.

When students can give a correct response quickly and easily, that is the time to ask them to explain their thinking, since usually the skill of talking about math lags behind the skill of doing math. This means that students of different abilities can be working on the same content area with the same questions: for some at a lower level, just doing the work is the job at hand; for others at a higher skill level, articulating the process is the job that requires work and keeps the interest high.

When students are familiar with the process, ask a student to "be the teacher" and read the instructions for other students to follow. The challenge of checking that student responses are correct, while running the group process and noticing patterns, will be interesting math experience for some students whose skills in doing the math are more advanced than most of the group.

Process

Each day, ask students to go the board to read and write some fractions. Encourage them to make themselves at home, to look around at other students' work, to stand beside someone who is good at math, to get a long piece of chalk, and so on. They should do whatever they need to be comfortable at the board. Especially, make sure there is an eraser between every pair of students. If a student has to ask for an eraser, he calls attention to his error. We want him to take a risk, so make sure he can quickly and quietly erase his answer if it is wrong.

Every day, the work follows the same pattern:

1. As you call out the first instruction, students will write the fraction(s).
2. Quickly look around and make sure that every response is correct, helping individuals as necessary.
3. Ask students to look around to see if their answer is the same as everyone else's. Different answers may be correct, and provide an opportunity to talk about the process.
4. Repeat for each of the bulleted instructions on the day.

5. Thank students for their participation and their excellent work, and invite them to sit down.

Day 1

- Think about all the people in the room. What fraction are men? Write that fraction. Write the fraction of people who are women. What fraction are teachers? What fraction are students?
- Think about the people at the front board. What fraction are men? Write that fraction. Write the fraction of people at the front board who are women.
- Think about the people at the side board. What fraction are men? Write that fraction. Write the fraction of people at the side board who are women.
- Suppose there were twelve people in the room and half were women. Write a fraction with 12 on the bottom that shows the fraction of women in that room. Correct. 6/12 is equivalent to 1/2.
- Suppose there were 100 people in the room and half were men. Write a fraction with 100 on the bottom that shows the fraction of men in that room. Correct. 50/100 is equivalent to 1/2.
- Write five fractions equivalent to 1/2.
- I'll give you three fractions, 1/2, 1/10 and 1/4. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Thanks. You may sit down.

Day 2

- Think about all the people in the room. What fraction are wearing glasses? Write that fraction. Write the fraction of people who are not wearing glasses.
- Think about the people at the front board. What fraction are wearing glasses? Write that fraction.
- Think about the people at the side board. What fraction are wearing glasses? Write that fraction.
- Repeat until everyone can write a fraction easily, asking for the fraction of people wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of students (for example, front board and side board), so that the denominators are not the same all the time. Sometimes include yourself, sometimes not, for example, "What fraction of the people in the room are wearing glasses? What fraction of the students are wearing glasses?"
- Suppose there were six pieces of pizza left after the party, and half had mushrooms on them. Write a fraction with 6 on the bottom that shows the fraction of pieces that had mushrooms. Correct. 3/6 is equivalent to 1/2.
- Write five other fractions equivalent to 1/2.

- I'll give you three fractions, 1/6, 1/2 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you three fractions, 5/8, 1/8 and 3/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Thanks. You may sit down.

Day 3

- Think about the people in this room. What fraction of the women are wearing black shoes today?
- What fraction of the men are wearing black shoes today?
- What fraction of the students are wearing black shoes today?
- Repeat until everyone can write a fraction easily, asking for the fraction wearing shorts, wearing skirts, wearing black, or wearing watches, etc. Ask the question of the whole room, and then of one or more smaller groups of students, so that the denominators are not the same all the time.
- Write five fractions equivalent to 1/2.
- I'll give you five fractions, 1/6, 1/2, 1/4, 1/12 and 1/100. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 4/7, 3/7, 1/7 and 7/7. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- (Use this question after you have taught a process for generating equivalent fractions.) Write a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /8; 3/4 = /8; 1/3 = /6; 2/3 = /6.
- Thanks, you can sit down.

Day 4

- Ask them to write several fractions describing people in the room, e.g., What fraction of the men have their hair tied back? What fraction of students have their hair tied back? What fraction of women?
- Write five fractions equivalent to 1/2.
- I'll give you five fractions, 1/10, 1/2, 1/4, 1/5 and 1/20. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 4/10, 3/10, 5/10 and 9/10. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.

- Write some equivalent fractions: a fraction equivalent to 1/2 with a bottom number of 4. (Write the question up in the usual way so that people can see it. 1/2 = /4). Continue with a few more: 1/4 = /12; 3/4 = /16; 1/3 = /9; 2/3 = /12.
- I eat 3/4 of a chocolate bar. What fraction is left?
- The kids eat 5/8 of the pizza. What fraction is left?
- The team plays the first quarter of the basketball game. How much of the game is left?
- I do 1/3 of my homework. What fraction is left?
- Thanks, you can sit down.

Day 5

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 1/4, 7/8; 5/10, 3/4, 5/6, 1/12, 3/100.
- Write five fractions equivalent to 1.
- I'll give you five fractions, 1/3, 1/10, 1/5, 1/9, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 89/100, 39/100, 51/100 and 9/100. Write them in order from smallest to largest. If you like, draw some diagrams.
- Write some equivalent fractions, 1/4 = /12; 1/3 = /12; 2/3 = /12; 3/4 = /12, 5/6 = /12.
- I watch 3/4 hours of a one-hour program. What fraction is left?
- I eat 1/2 of an apple. What fraction is left?
- The kids use up 4/5 of the toothpaste. What fraction is left?
- I watch the first half of the soccer game. How much of the game is left?
- I have to read 10 pages for homework. I read 9 pages. What fraction of my homework is left?
- Reduce these fractions to lowest terms: 2/4, 4/8, 6/12, 8/16, 12/24.

Day 6

- Draw three columns at the board. At the top of one write "less than 1/2;" at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 5/10, 4/8, 1/100, 6/12, 9/14, 10/20, 3/6.
- Write five fractions equivalent to 1.

- I'll give you five fractions, 1/5, 1/14, 1/7, 1/4, and 1/2. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- I'll give you four fractions, 5/8, 1/8, 3/8, and 8/8. Write them in order from smallest to largest. If you like, draw some diagrams or use the math tools.
- Write some equivalent fractions: 1/4 = /16; 1/3 = /15; 2/3 = /15; 3/5 = /20; 5/6 = /24.
- I watch 1/4 hour of a one-hour program. What fraction of the program is left?
- We eat 2/3 of a pizza. What fraction of the pizza is left?
- I drive 9/10 of the way to Nanaimo. What fraction of the trip is left?
- I have five hours of homework to do. I work for three hours. What fraction of the work is left?
- The Yankees played 5 innings against the Red Sox. What fraction of the game is left?
- Reduce these fractions to lowest terms: 4/10, 2/12, 4/14, 6/9, 12/16, 7/21.

Day 7

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 4/5, 1/8, 91/100, 8/15, 7/14, 5/10, 6/6.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- I'll give you three fractions, 1/6, 9/10 and 2/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1/2, or equivalent to 1/2.
- I'll give you five fractions, 5/9, 1/9, 7/9, 2/9 and 9/9. Write them in order from smallest to largest.
- Write some equivalent fractions: 1/4 = /20; 2/3 = /18; 3/5 = /15; 3/8 = /16; 5/9 = /18.
- I drink 1/2 of my cup of coffee. What fraction is left?
- We walk 3/4 of a block. What fraction is left?
- There were 10 questions on the test. I got 7 right. What fraction did I get wrong?
- The teams played one period of hockey. What fraction of the game is left?
- Reduce these fractions to lowest terms: 4/20, 2/28, 5/15, 6/15, 5/35, 7/35.
- Add: 3/4 + 1/4 ; 1/8 + 3/8; 4/9 + 3/9.

- Subtract: 1 – 1/2; 1 – 3/4; 1 – 2/3.

Day 8

- Draw three columns at the board. At the top of one write "less than 1/2," at the top of the next, write "= 1/2," and at the top of the last column write "more than 1/2." I'll give you some fractions. You write them in the correct column: 5/8, 1/21, 91/100, 7/13, 6/12, 9/18, 3/3.
- Write five fractions equivalent to 1.
- Write five fractions larger than 1.
- Write five fractions smaller than 1.
- Draw three columns. At the top of one write "less than 1," at the top of the next, write "= 1," and at the top of the last column write "more than 1." I'll give you some fractions. You write them in the correct column: 5/8, 4/4, 9/10, 3/2, 7/7, 4/1, 6/6.
- I'll give you three fractions, 1/6, 10/10 and 6/4. Write them in order from smallest to largest. Think about whether they are bigger or smaller than 1, or equivalent to 1.
- I'll give you five fractions, 5/3, 1/3, 2/3, 3/3 and 10/3. Write them in order from smallest to largest.
- Write some equivalent fractions: 1/4 = /16; 2/3 = /21; 3/5 = /40; 3/8 = /32; 5/9 = /45.
- The third quarter of the football game is over. What fraction of the game is left to play?
- Four team mates run a relay race. What fraction of the race does each person run?
- Reduce these fractions to lowest terms: 4/10, 7/21, 8/16, 6/18, 5/25, 7/56.
- Add: 1/2 + 1/2; 1/2 + 1/4; 1/2 + 1/8.
- Subtract: 1/2 – 1/2; 1/2 – 1/4; 1/2 – 1/8.

Following days

Many days of board work follow this pattern—adding and subtracting, multiplying and dividing fractions, continuing to build up a sense in students of the relative size of fractions and mixed numbers.

Math Song Book

Velma McKay, Access Education Math Instructor, College of the Rockies, Cranbrook, BC, has given permission to reprint these excerpts from her math songbook. She wrote the words, except as noted. If you would like more songs from her collection, you can reach her at **mckay@ cotr.bc.ca** (Write this address with no spaces.)

Math Phobia

(Tune: Hey Jude)

Hey you, Don't be afraid
Math can't hurt you
It's really not that bad
The minute you let it
into your heart
then you will start
to learn it better.
Learning Math, can be fun

Math is fun, Hey you
Learning Math, can be fun
Math is fun, Hey you…

Math Song Quest

(Tune: Impossible Dream)

This is my quest
To teach you math songs
No matter how silly,
No matter how long
To teach you them right, without question or pause
to be willing to
sing them out loud
for a very good cause
And I know if you'll just learn them too,
then the formulas
will be easy for you

And the world will be better for this
if math songs
silly though they may be,
help students
be more successful
then its all worth it to me
This is my quest to teach you math songs

to teach you math songs…

Silly Math Songs

The lyrics are a collaboration. My original math lyrics inspired George Rhys of College of the Canyons, in Santa Clarita, California to find a midi version of the McCartney tune, modify the lyrics, and build a delightful power point presentation for this song, complete with midi backup and an explanation for the logic behind the use of math songs. (Think mathematical karaoke.)

(Tune: Silly Love Songs-McCartney)

Some people have had enough of silly math songs,
I look around me and I see it isn't so, oh no
Some people want to fill the world with silly math songs
And what's wrong with that?
I'd like to know?
'Cause here I go again…
I love math! I love math!
I-I can't explain
the math is plain to me
Oh can't you see
when I sing the songs it helps my memory
Oh can't you see.
Math doesn't come in a minute
Sometimes it doesn't come at all
I only know that when I sing it
I just remember it, I can remember it ALL!
I love math! I love math!
How can I get you to sing my math songs…

I'll Sing You Math Songs

(Tune: What would you do if I sang out of tune?)

What would you do
if I sang out of tune?
Would you stand up and walk out on me?
Lend me your ears
and I'll sing you math songs
and I'll try not to sing out of key.
I use, math songs to remember the rules
I use, math songs to remember the tools
What do you need to remember?
Try singing it i-in a song?
It could help you remember
Just singing math songs

You Can't Divide by Zero

(inspired by Peter Gzowski interview with a singing math teacher from Ontario, second verse by Peter Robbins, Kwantlen University College, modified by me.)
(***Tune : Rudolf the Red Nosed Reindeer)***
Zero's a lovely number.
It will multiply and add just fine,
But you can't divide by zero
'Cause the answer's undefined.
All of the other numbers
Multiply and divide with ease
but if you divide by zero
all your calculator says is E

Perimeter Song

(Tune: O Christmas Tree)
The perimeter of a rectangle
is two lengths plus two widths.
The perimeter of a triangle
is the sum of all the sides.
The perimeter of a circle
is also called the circumference.

I is P R T (I is Pretty)

(Tune: I feel pretty-from Westside Story)
I is P R T (I is pretty)
Interest equals
Principa-al times ra-ate times time
I is P R T (I is pretty)
Calculates simple inter-est just fine.

Percent, Decimal and Fraction Notation

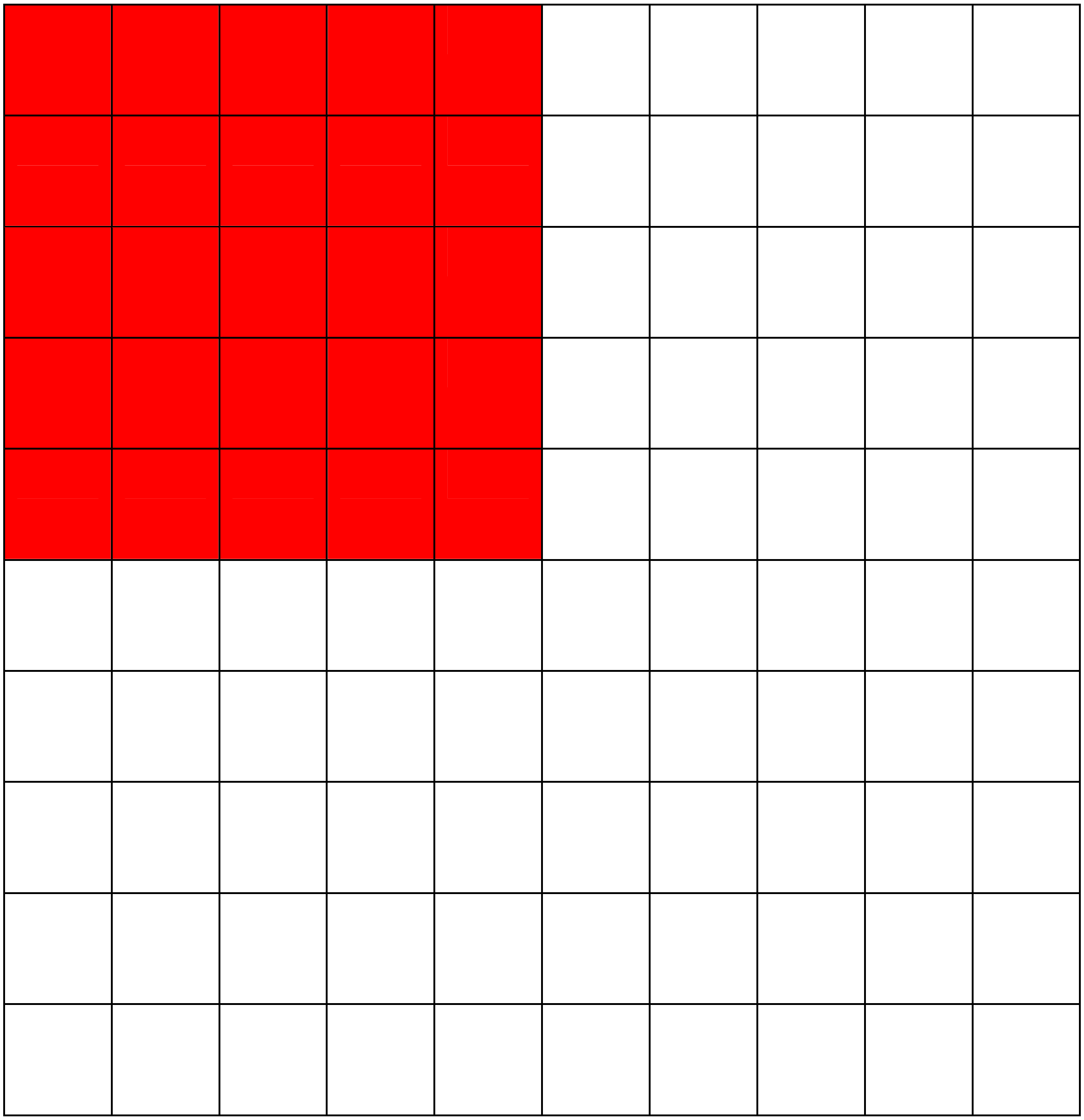
Activity: Match the numbers to the chart

1. Copy the charts on the next few pages. The charts are arranged in order of difficulty. (The last chart is blank for you to fill in as needed.)
2. Cut the numbers in strips from the bottom of each page; keep the strips with each chart. You will need a strip for each student in your class.
3. Choose the number of charts and accompanying strips so that everyone in the class will get one strip. Choose the appropriate level of difficulty.
4. Range the charts along the ledge of the blackboard, or on a bulletin board. Shuffle the strips, place them face down and ask each student to pick one strip.
5. Each student should take his strip to the chart that it matches; when the three students meet at the chart, they should confirm with each other that their strips show equivalent amounts, and prepare to explain their reasoning to the class.
6. Ask each group of students to explain their thinking to the class.

***Variation*:** Hand out only two of the strips from each chart, for example, the decimal and the fraction notation. When the two students meet at the chart, they should confirm that their strips show equivalent amounts, and then generate the missing notation, in this example, the per cent notation.

***Working with a single student*:** Use three of the charts at an appropriate level of difficulty; shuffle the strips that go with the charts, and ask the student to match them and explain their thinking.

Again, you could use only two of the strips from each chart and ask the student to generate the third notation.



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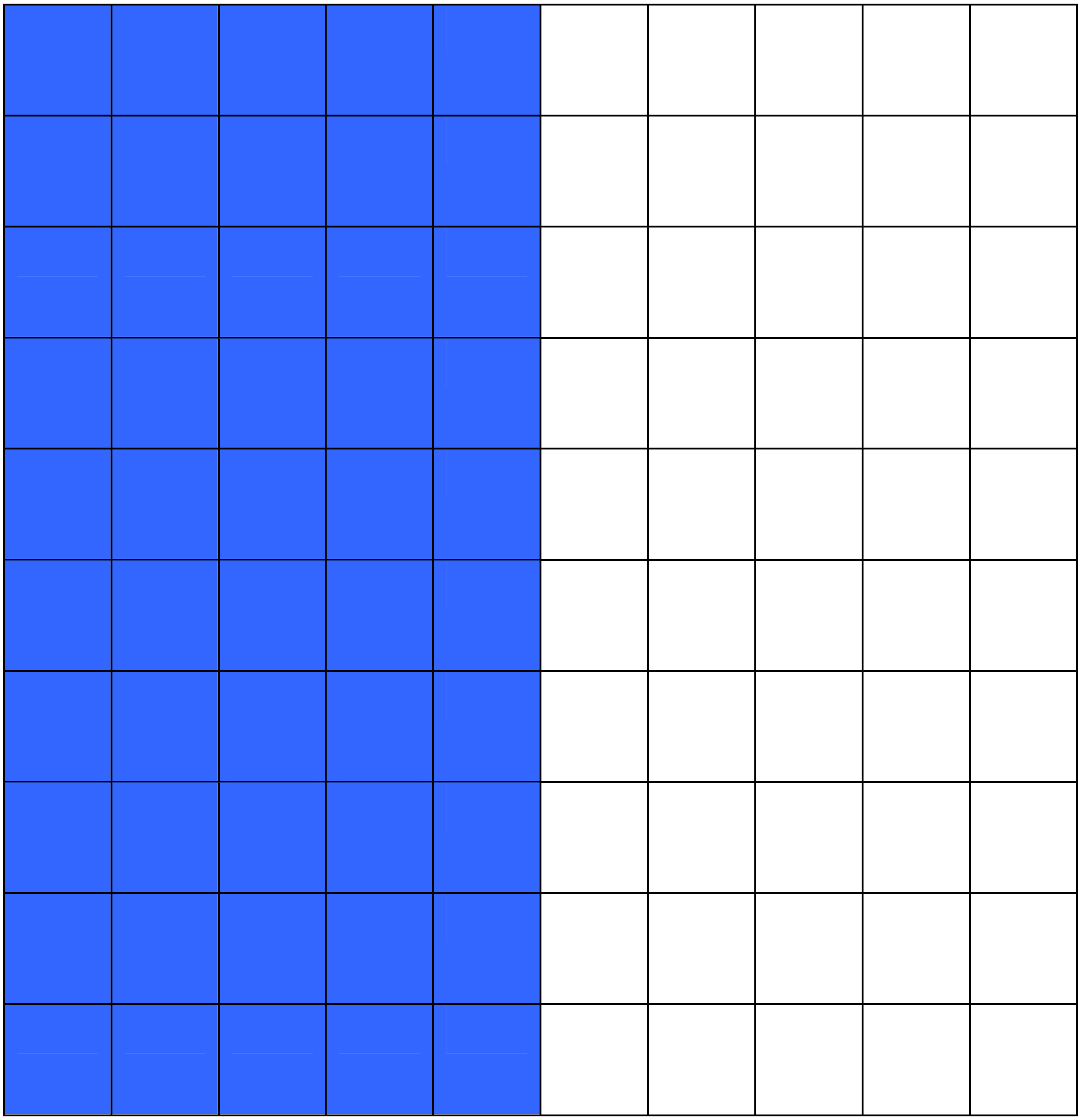

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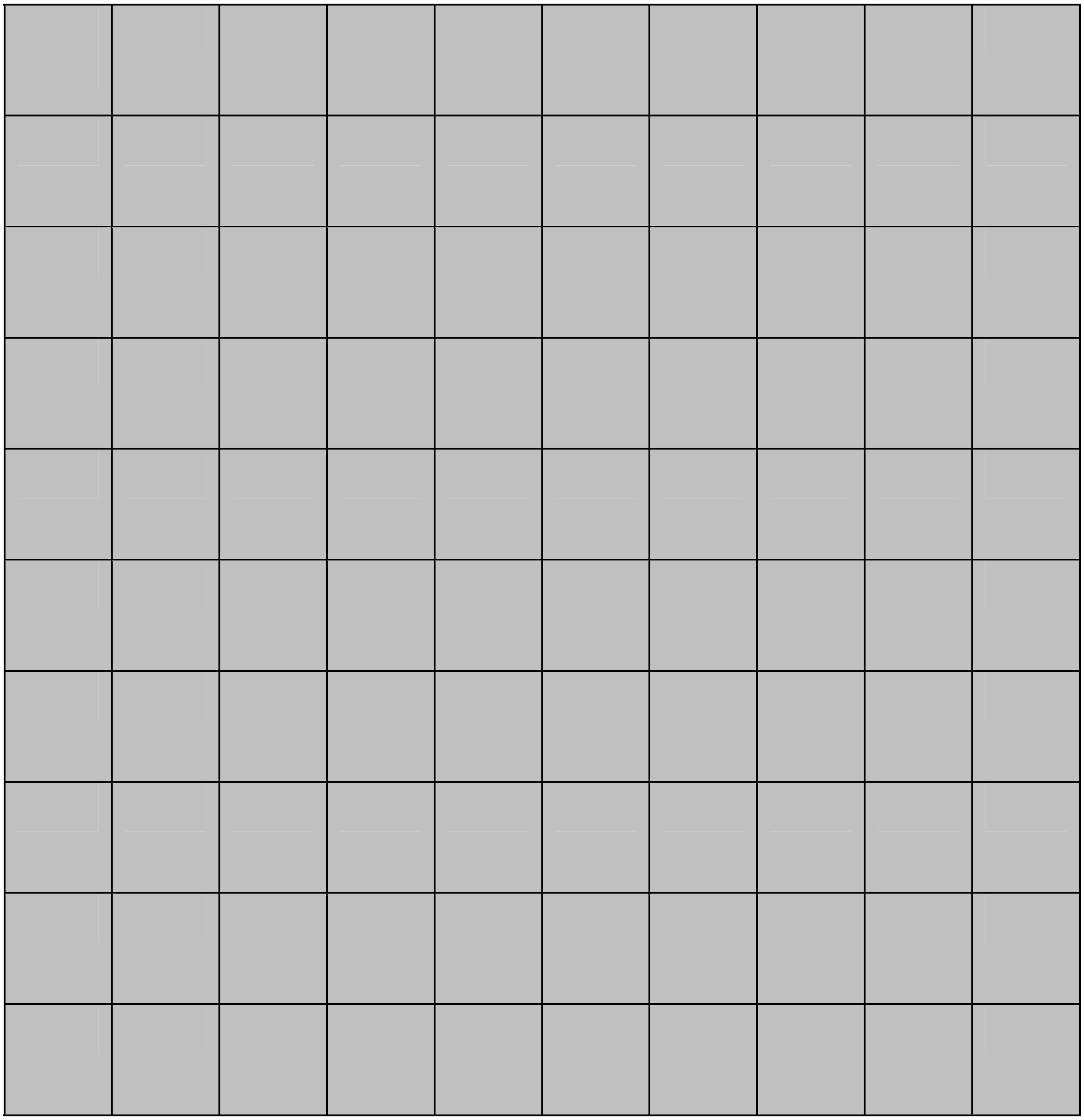

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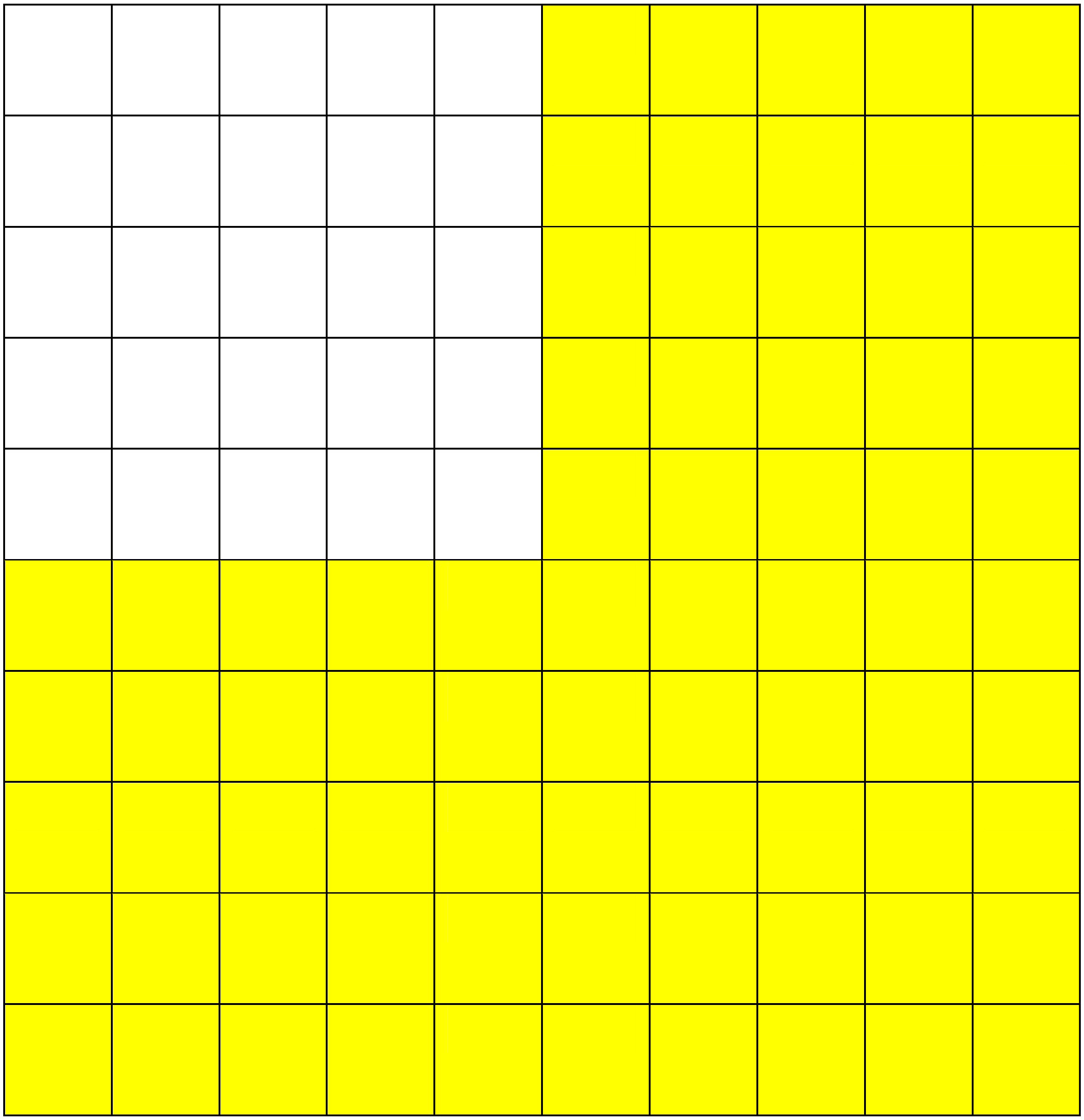

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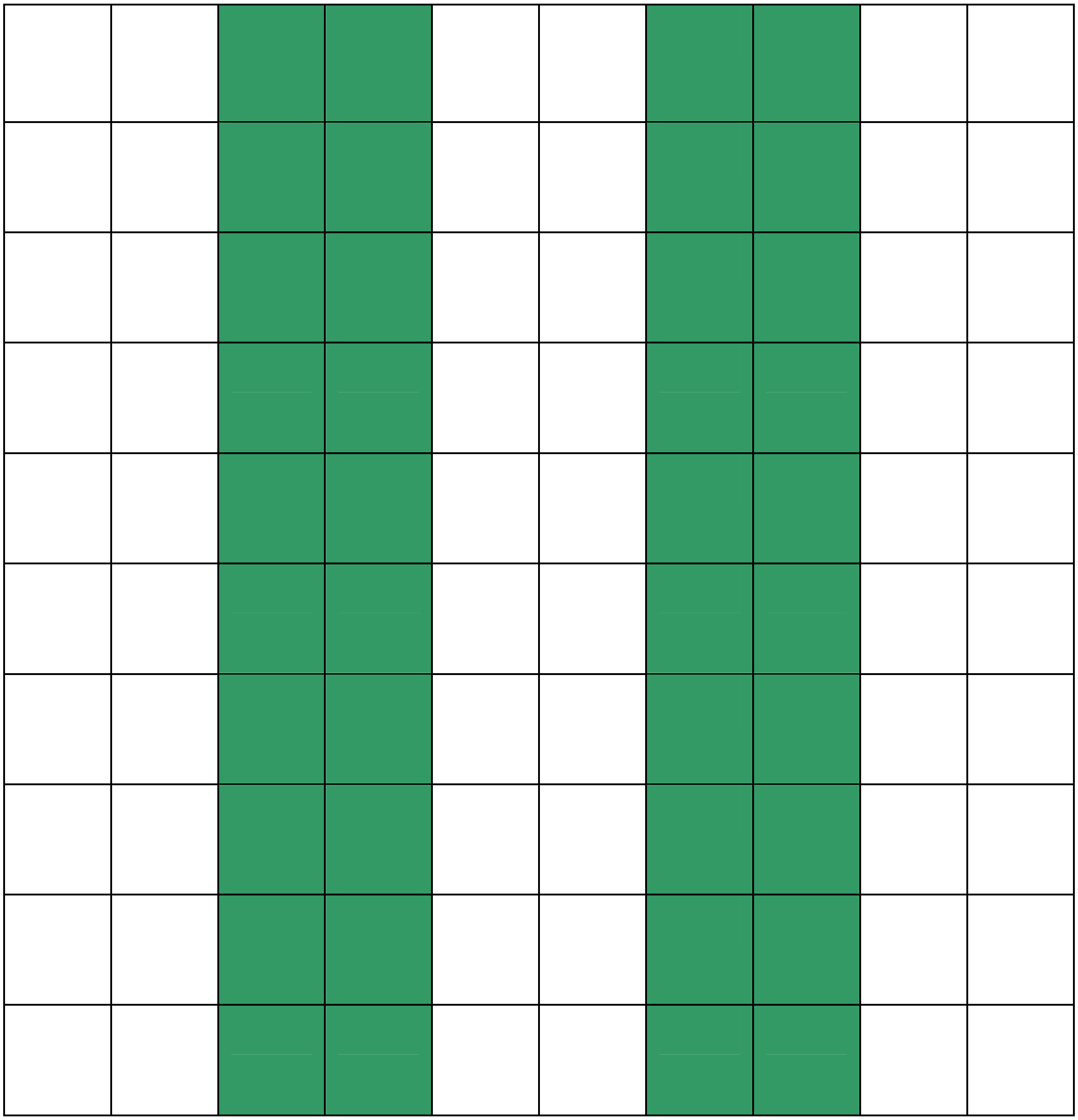

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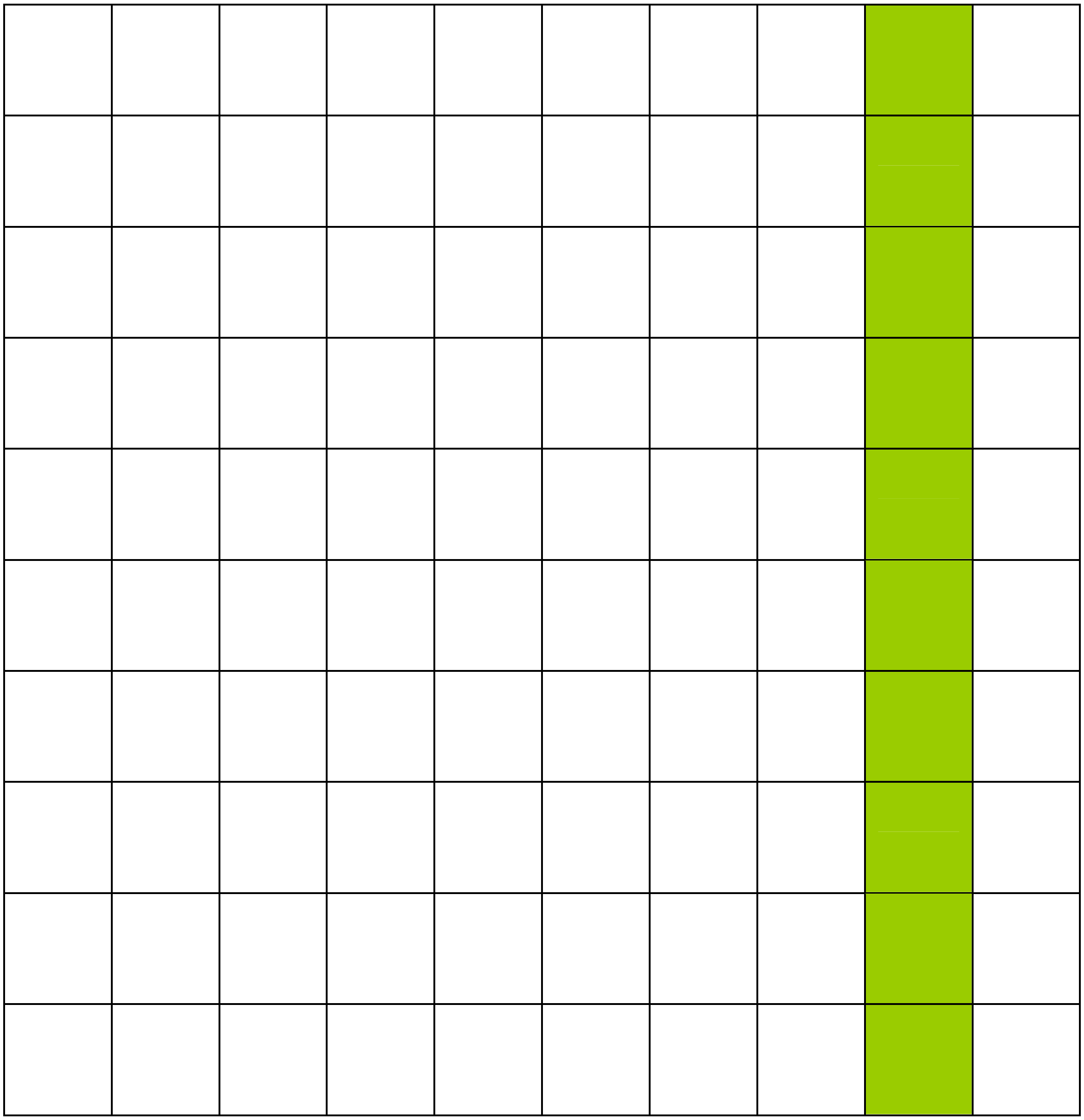

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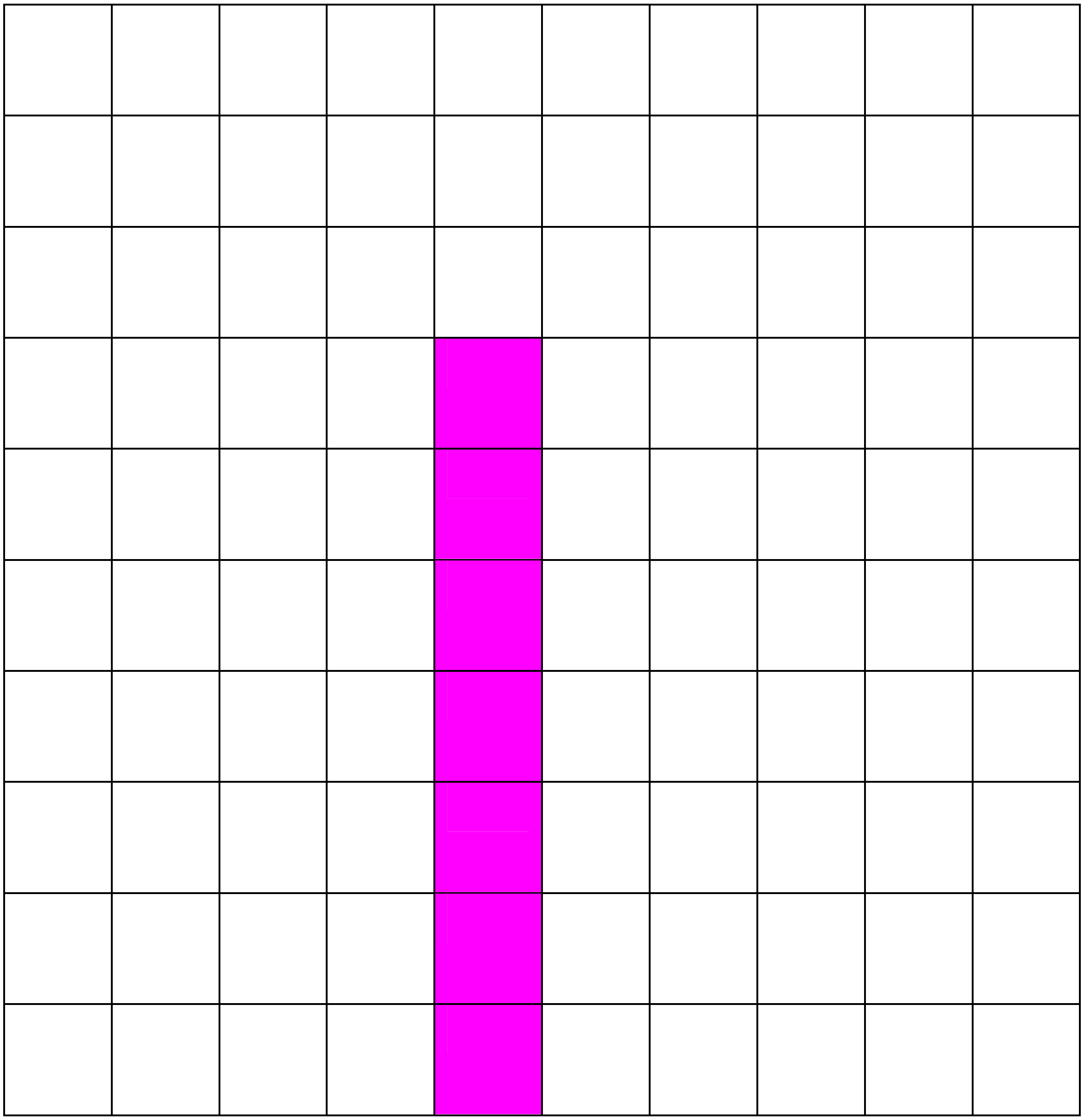

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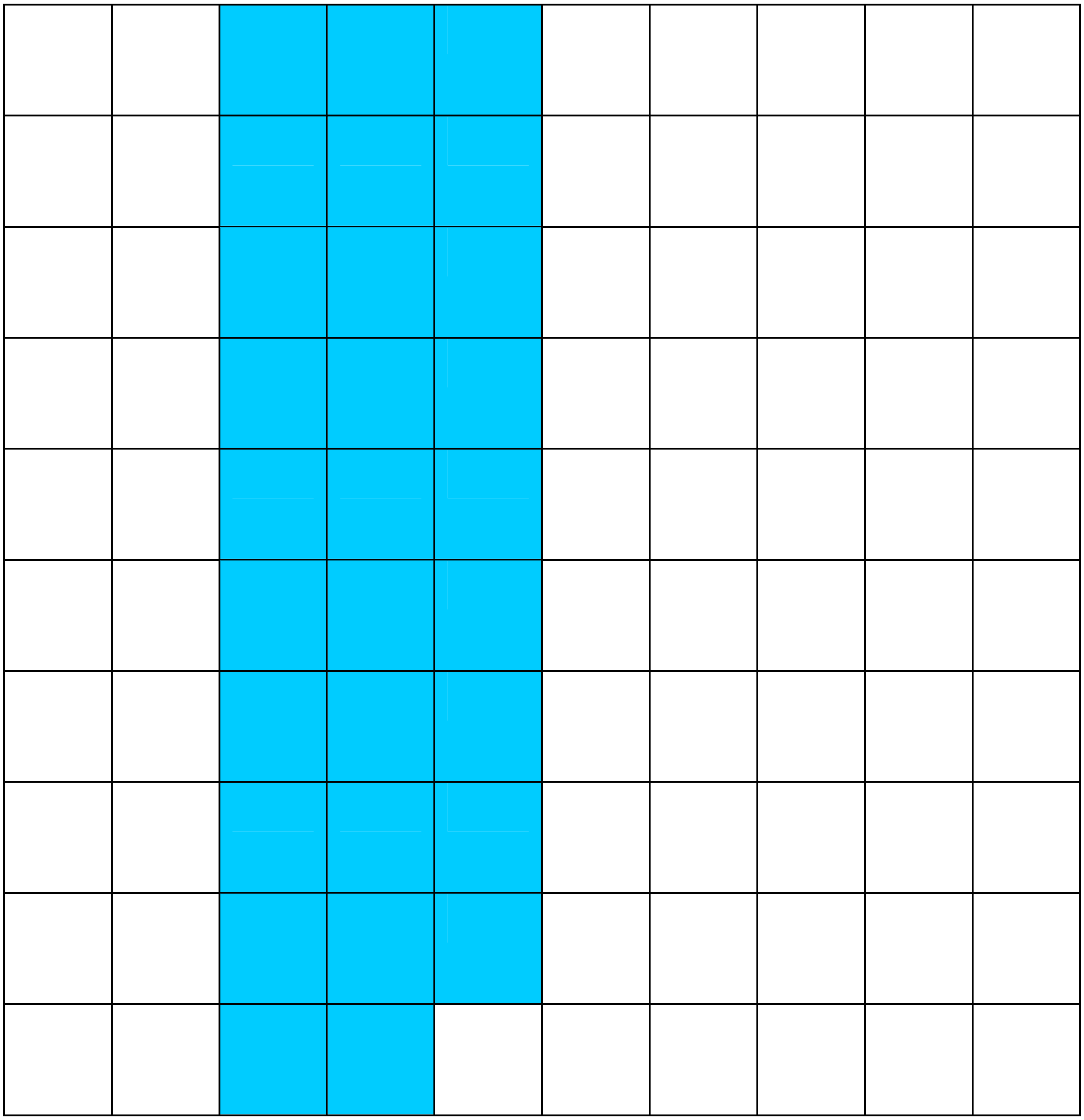

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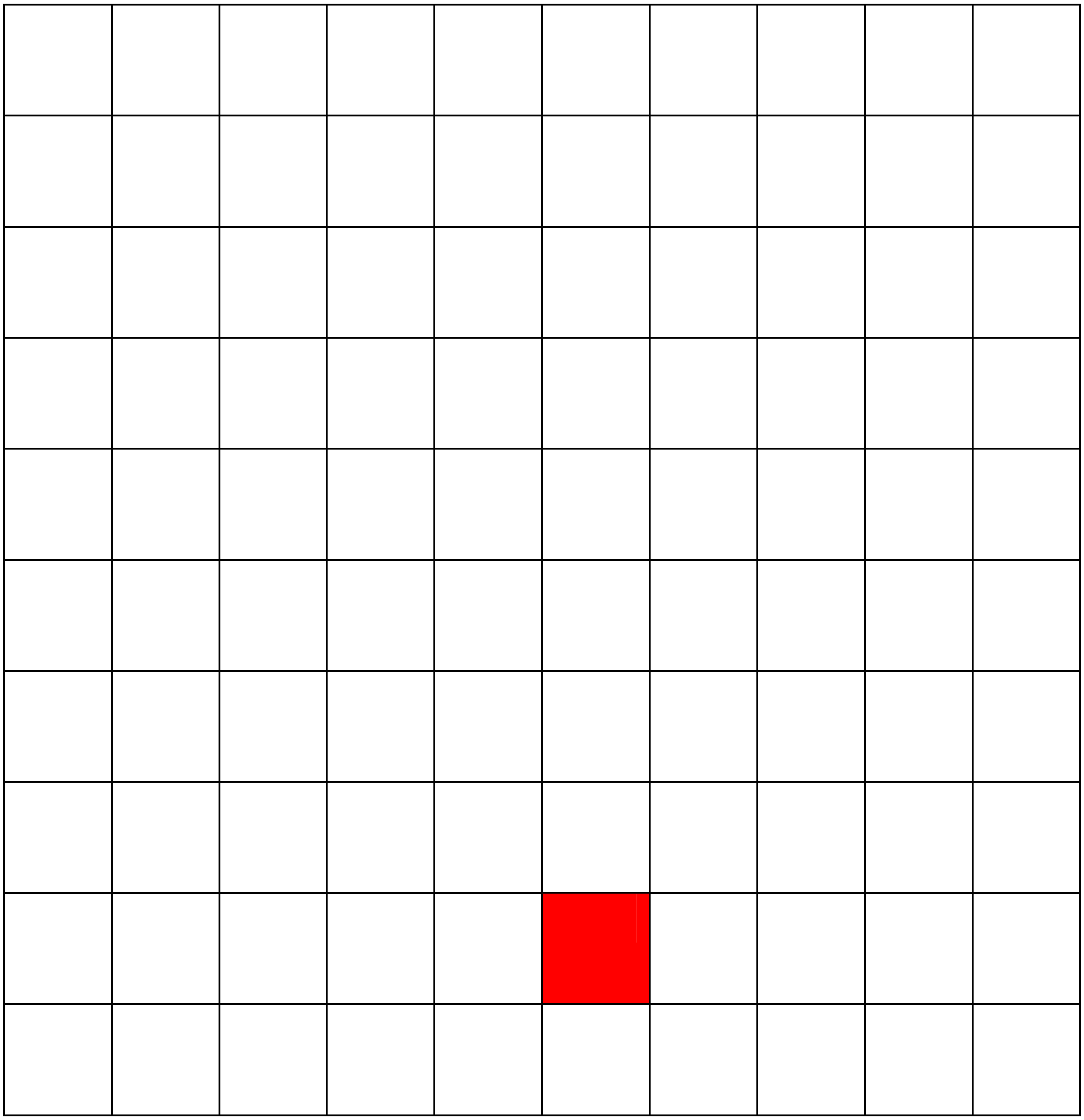

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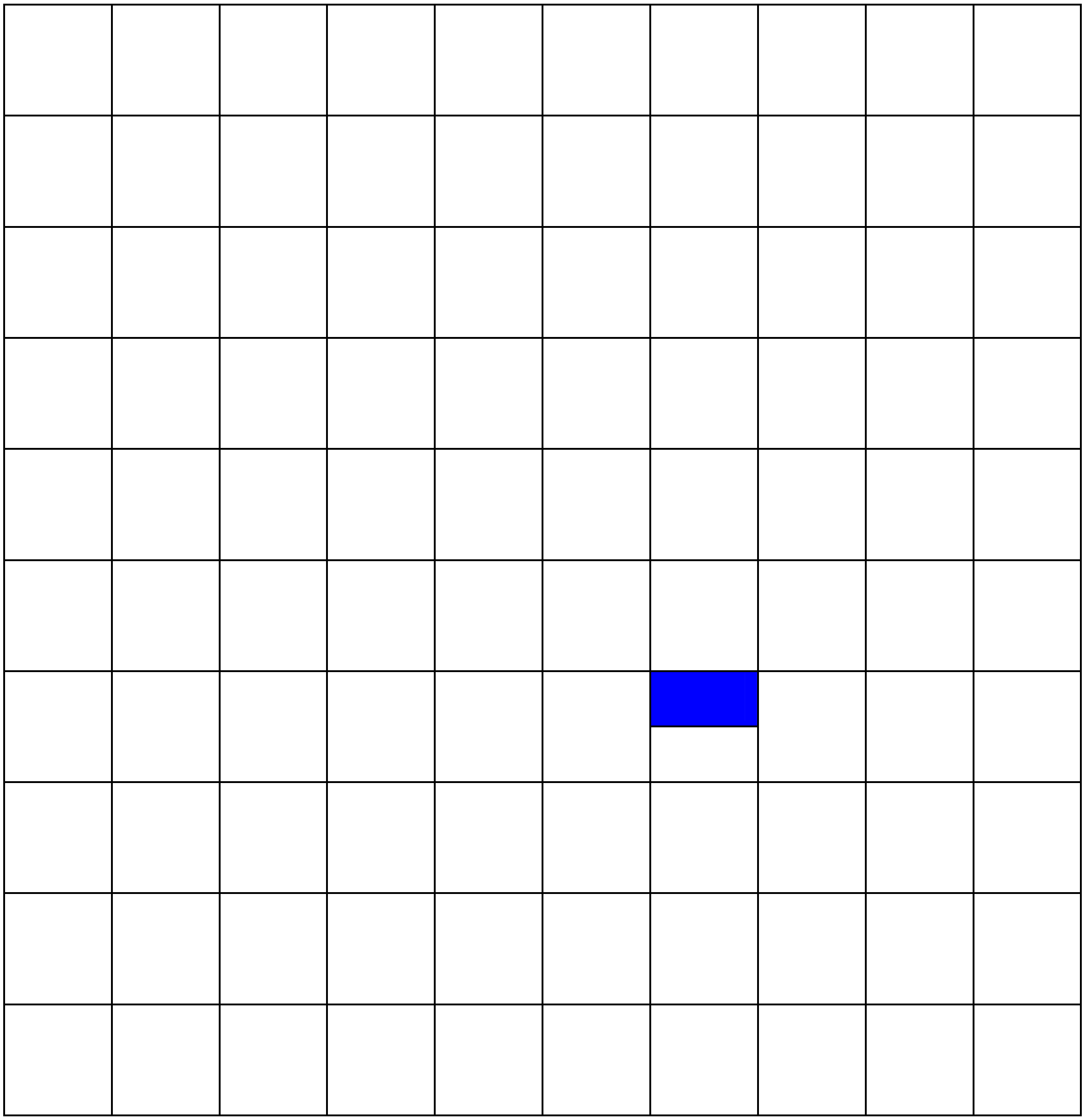

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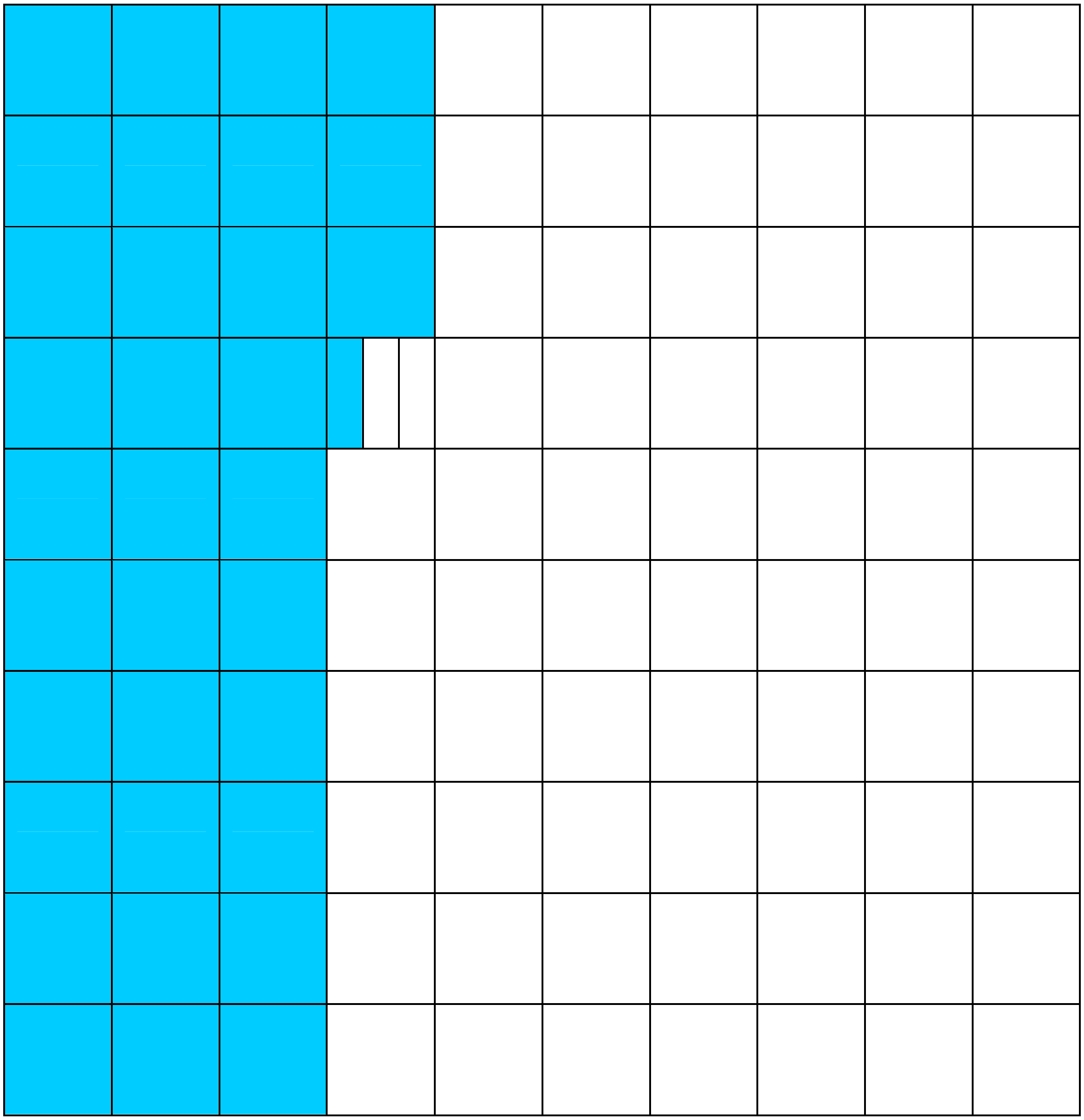

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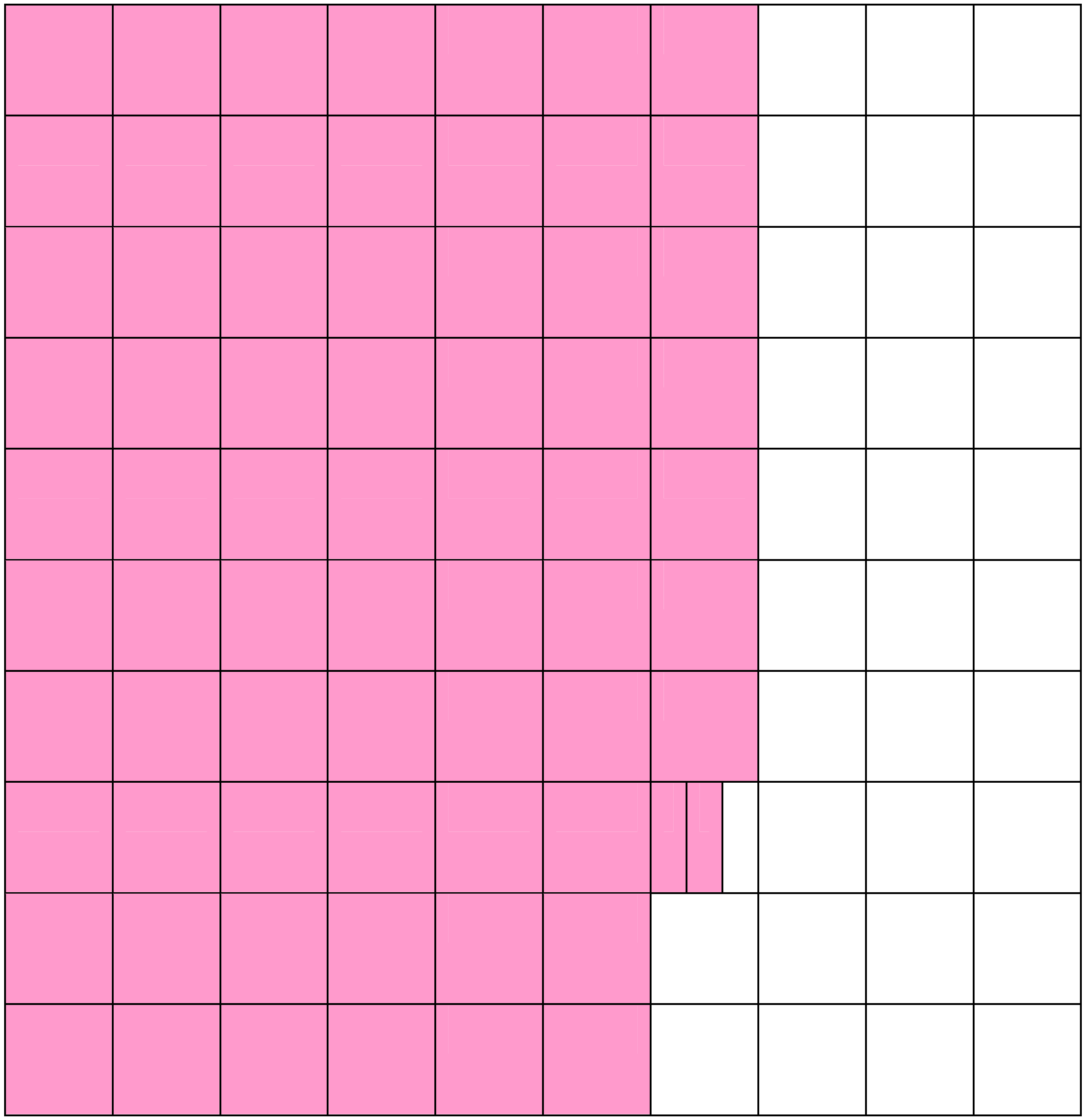

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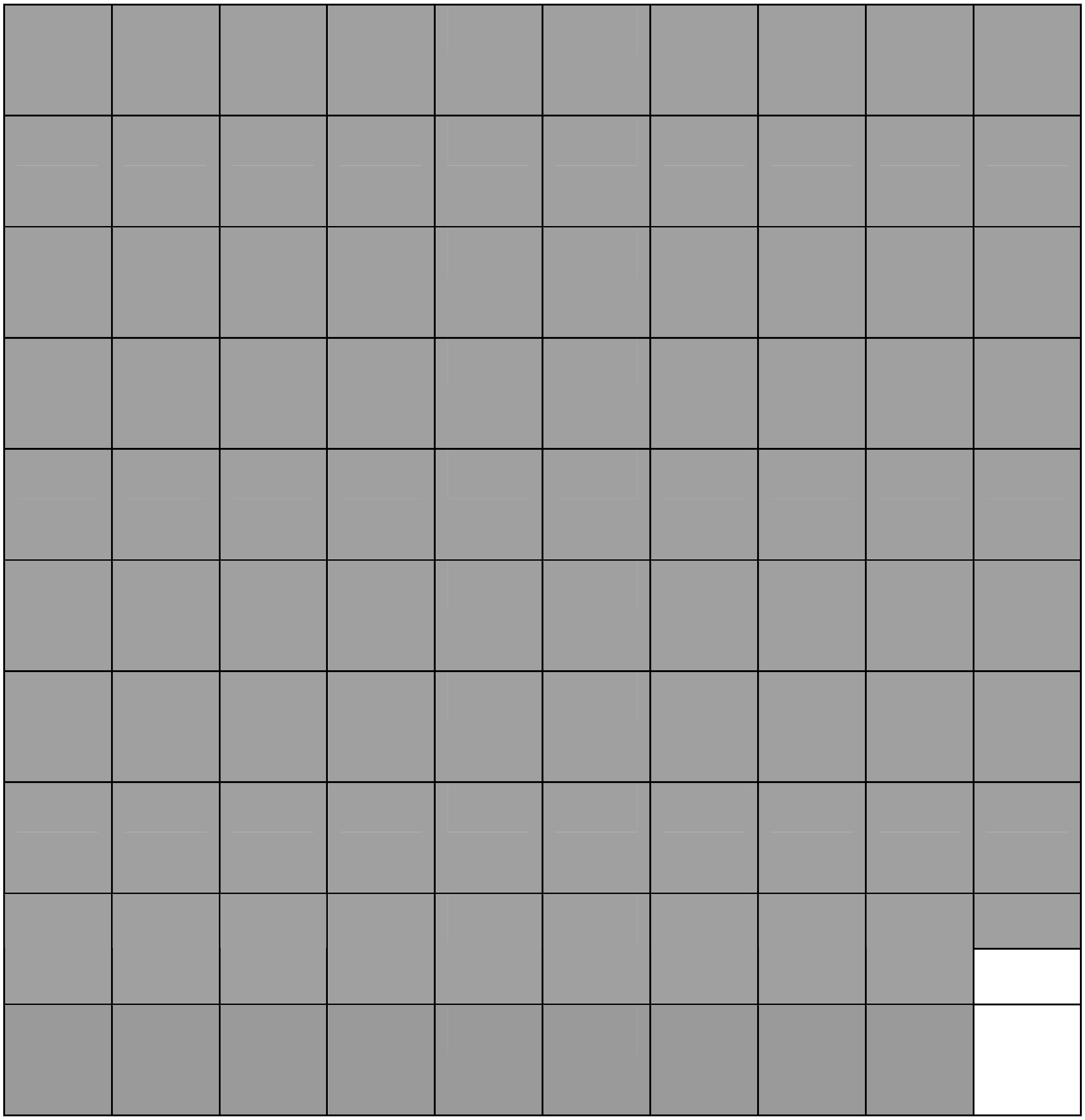

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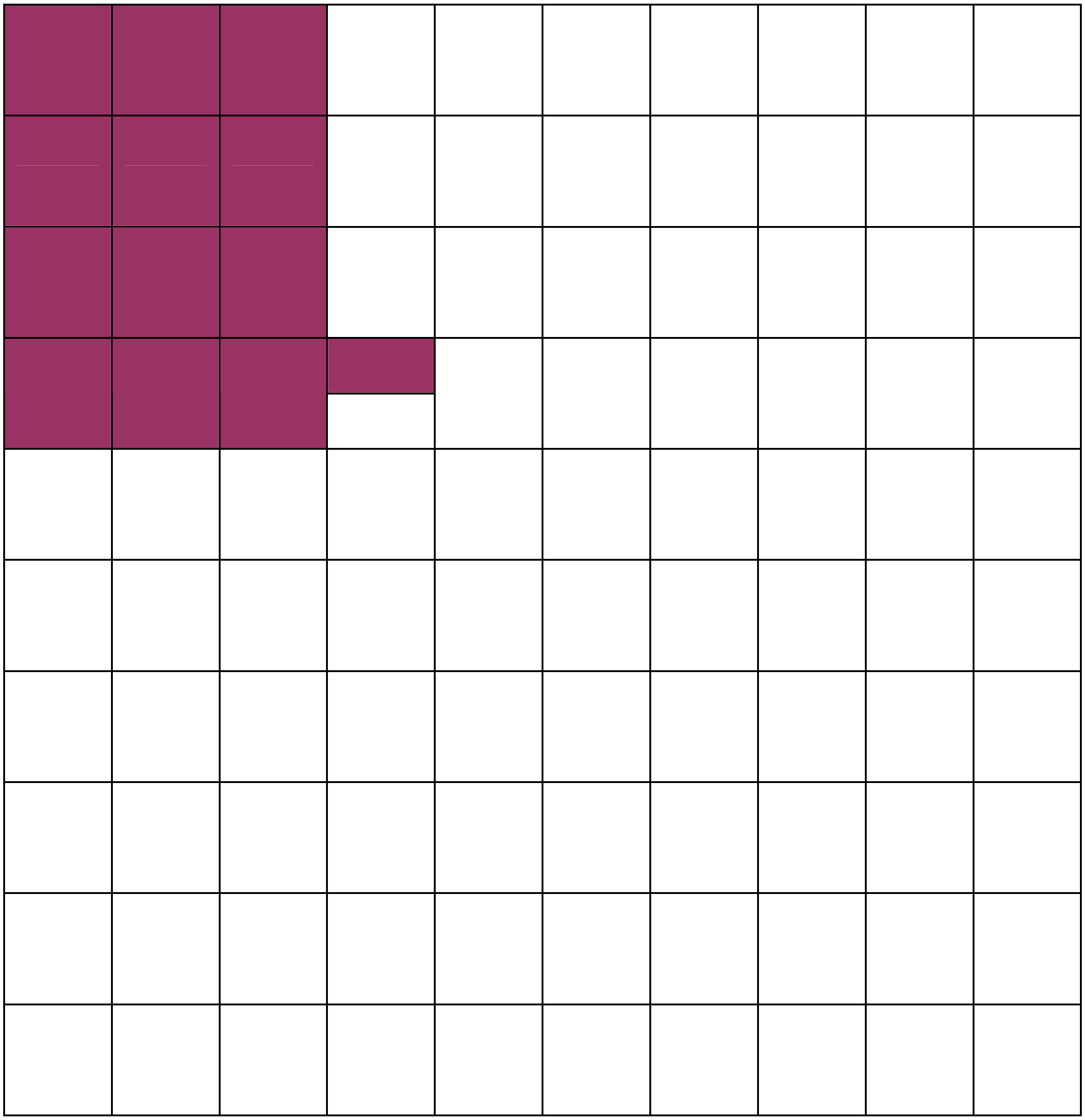

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Websites of Interest

Professional Associations

Adult Numeracy Network http://www.literacynet.org/ann/

Adults Learning Mathematics http://www.alm-online.org/

Sources of Information

Canada: National Adult Literacy Data Base www.NALD.ca

Australia: http://www.saalt.com.au/numeracy/anamol/index.htm

United Kingdom: http://www.literacytrust.org.uk/Database/adultres.html

USA: National Institute for Literacy http://www.nifl.gov/

Practitioner Training

Ottawa-Carleton District School Board.
http://www.lbspractitionertraining.com/html/AboutDAN.html

On-line Activities for Students

National Center for Education Statistics. Create a graph.
http://nces.ed.gov/nceskids/createagraph/

BBC Skillswise. http://www.bbc.co.uk/skillswise/

Multiple Intelligences test. Gives a graphic print out.
http://www2.bgfl.org/bgfl2/custom/resources_ftp/client_ftp/ks3/ict/multiple_int/index.htm

Learning styles. Gives a graphic printout http://www.learning-styles-online.com/inventory/default.asp?ref=ga&data=learning+styles+free+test

Learning styles. A checklist shorter than the one above.
http://www.metamath.com/lsweb/dvclearn.htm

Miscellaneous

Learning and Violence. www.learningandviolence.net

National Library of Virtual Manipulatives. http://nlvm.usu.edu/en/nav/category_g_2_t_1.html

Junior Undiscovered Math Prodigies. http://www.jumpmath.org/

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